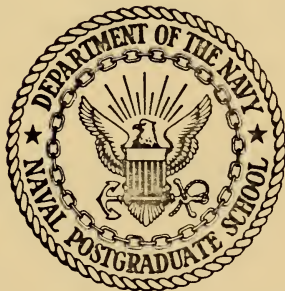


A PROBABILISTIC EVENT-STEP COMPUTER SIMULA-
TION OF A REPAIRABLE ITEM INVENTORY
SYSTEM

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THESIS

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SIMULATION OF A REPAIRABLE ITEM
INVENTORY SYSTEM

by

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March 1972

Approved for public release; distribution unlimited.

A Probabilistic Event-Step Computer Simulation of a Repairable
Item Inventory System

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ABSTRACT

A probabilistic event-step computer simulation of a repairable item inventory system with exponential inter-arrival times between demands is constructed. The model allows for a wide variety of repair times and lead times to be considered. The major parameters are investigated for their sensitivity to system changes in order to help the user evaluate given or proposed inventory policies and parameters. A complete description of the model is presented and various measures of supply performance such as the expected number of unit years of backorders per unit period and the fill rate are provided. All flow charts and the GPSS/360 program listing are included.

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TABLE OF SYMBOLS AND ABBREVIATIONS

LFN	Location field name; a mnemonic symbol used to identify a particular program entity, section, or block in the simulation. Such names are primarily used for ease in programming and model identification.
XHj	Halfword savevalue j; $j=1,2,3,\dots,100$
A	XH72, instantaneous number of items on order.
ASO	Aviation Supply Office
B	XH71, instantaneous number of backorders.
BARB	XH96, the expected number of items on backorder at an arbitrary point in time multiplied by 100.
BKORD	Acronym for the Backorder Event
B_T	XH97, expected number of unit years of backorders, for 10 years of operation multiplied by 100.
B_T	Expected number of unit years of backorders, $(B_T/10)$.
BTCHZ	LFN for the table which stores the value of the size of each batch of items repaired.
BTIME	LFN for the table which stores the length of time each backorder is outstanding.
C	XH73, instantaneous number of items in repair department.
D	XH74, instantaneous number of items ready for issue.
DIFF 1	XH90, difference between the simulated result for the average number of items on hand at an arbitrary point in time and the theoretical expression for the same quantity multiplied by 100.
DIFF 2	XH89, difference between the expected number of unit years of backorders per unit period and the average number of backorders at an arbitrary point in time multiplied by 100.
ENOH	LFN for the table which stores the average number of items on hand for each random observation multiplied by 100.

F XH77, cumulative number of demands satisfied without any time delay (the number of fills).

FASTR LFN for the storage which represents the repair process for the infinite server queueing system.

FFCj Function follower card j, $j=1,2,3,4$.

FILRT XH69, fill rate times 10,000, i.e., $7356=73.56\%$.

FSTRQ LFN for the queue which represents the entire repair department for the infinite server queueing system.

IP XH70, instantaneous value for inventory position.
 $IP=(D-B+C+A)$

K XH4, maximum batch size for single server queueing system.

L XH3, minimum batch size for single server queueing system.

LBL XH14, lower bound for uniform lead time distribution.

LBR XH9, lower bound for uniform repair time distribution.

LFN Location Field Name, a mnemonic symbol used to identify a particular program entity, section, or block in the model. Such names are primarily used for ease in programming and model identification.

M XH79, cumulative number of demands in a unit period.

MOPj Measure of Performance j, $j=1,2,\dots,12$.

NBARB LFN for the table which stores the number of back-orders for each random observation.

NUMC LFN for the table which stores the number of items in the repair department for each random observation.

ORDER Acronym for the Order Event.

P (XH5/1000) Probability of a type 1 item (repairable item), i.e., $XH5=0900 \Rightarrow P=.9$

Q XH1, reorder quantity.

R XH2, reorder point.

RDYRT LFN for the table which stores the value of XH44 for each random observation. $XH44=10,000$ if $D>0$, 0 if $D=0$. Used in ready rate calculation.

SLOWR LFN for the storage which represents the repair process for the single server queueing system. The simulated repair time.

SLOWQ LFN for the queue which represents the repair department for the single server queueing system. The simulated time the item is in the repair department.

SPCT Acronym for the Inspection Event.

TYPE 1 A failed item which is repairable. Occurs with probability P .

TYPE 2 A failed item which is not repairable. Occurs with probability $1-P$.

UBL XH15, upper bound for uniform lead time distribution.

UBR XH10, upper bound for uniform repair time distribution.

\bar{X} Mean demand during repair time.

$Y(j)$ Function values for empirical lead time and repair time distributions. $0 \leq Y_j \leq 1, j=1, 2, \dots, 10$

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I. INTRODUCTION

Traditional inventory analysis is ultimately concerned with two decisions: (1) when to order and (2) how much to order. Most of the work done in the past addresses itself to these questions for non-repairable items (consumables). A thorough treatment of such inventory models for both the backorders and lost sales case for numerous resupply and demand distributions now exists.

However, at least in military applications, repairable items have now been found to be of great importance both materially and financially. Repairables often account for a significant portion of the items controlled by an activity and even more significantly when their portion of the total dollar investment is considered. For example, focus on one of the inventory control points of the U.S. Navy dealing with repairables such as the Aviation Supply Office (ASO) in Philadelphia, Pennsylvania. At fiscal year's end 1971 repairables represented 18.7% of the different types of line items controlled by this activity for a total of 66,000 separate items. These same items reflect an investment of nearly 3.1 billion dollars, almost 60% of ASO's total dollar investment. Further, ASO spends about 16 million dollars per month for repairs on these items and another 28 million dollars monthly for procurement of such items. Among ASO managed repairable items, 365,000 units were reworked in

fiscal year 1971. In view of the above figures it is easy to see why a need exists to help the manager improve his control over repairable items. It appears that an effective control over repairs would free for other needs many ASO dollars now spent on procurement.

Until fairly recently the control of repairable items had not received much attention. Even in the case of consumables there are major difficulties in applying inventory theory to the problem because of the intricate relationships of the involved item parameters, such as demand and lead times. Also the number of different types of line items complicates the task. The introduction of repairable items makes the problem even more difficult since repairable items often require special storage, handling, packaging and shipping procedures due to their complexity. In addition, the introduction of the extra source of supply, the repair department, causes further complications. On the other hand, repairable items tend to minimize the number of different type of line items needed to be carried by an activity; the parameters are usually more easily determined than those of consumables; and the procurement lead times are more reliable. In addition, upper bounds for the total demand can often be determined, for if a given repairable item is a subcomponent of a larger unit, the number of larger units in use is generally easy to determine.

Few people have addressed the problem of repairables. However, Schrady [Ref. 8] and others at the U.S. Naval

Postgraduate School, as well as Feeney and Sherbrooke [Ref. 6], have looked at the problem for the deterministic case. Feeney's and Sherbrooke's model included the capability of handling compound Poisson demands but without degradation of supply. Wilson and Richards [Ref. 10] extended this model to allow for this possibility. Due to problems of convexity and others, no one has come up with a quick and easy way to evaluate and compare alternative repairable item inventory systems on a larger scale. Richards [Ref. 7] addresses this problem and this paper supplements his work by providing a detailed simulation model along with a programming guide and flow charts. This model enables the user faced with such problems to easily come up with numerical values for various measures of performance such as the average number of back-orders per unit period or the average number of fills per unit period. These measures of performance can then be used by the manager to evaluate and compare alternative systems. Further, the model can also be used as a teaching aide to help students and/or supply personnel get a clearer understanding of the relationships between the various parameters involved in various inventory systems. A more detailed description of the model and how to use it are given in sections II, V, and VI of this paper.

II. THE MODEL

The model presented in this paper is a general event-step computer simulation of the interactions between the major parameters for Q R, continuous review inventory systems. Here, Q is the order quantity and R is the reorder point for a given system. The purpose of this general model is to test and investigate the sensitivity of the parameters involved to help the decision maker evaluate, modify, and/or set up appropriate inventory systems. It is also used to demonstrate the analytical results obtained by Richards [Ref. 7]. The inventory systems simulated are single item, single echelon, and repairable or non-repairable systems where failures are assumed to be Poisson distributed. The model is a probabilistic simulation in that the success or failure of any probabilistic event is determined in the model by comparing the numerical value assigned to the probability of success or failure to a program generated random number. In this model the numbers so used are obtained from many different distributions, using the one which best fits the particular parameter being considered in each case. Further, it is an event-step model in that the master program is controlled by a subprogram called an event list which is repeatedly updated as the master program switches from one event subroutine to another. This eliminates the necessity to look at the status of the entire system after each time unit (one hour) to decide whether or not an event has occurred.

Figure 1 shows the general type of inventory systems addressed in this paper. In order to help the reader understand the relationships pictured there, the following definitions for the on-hand inventory, repair inventory, and others are given. Note that because of the addition of the repair facility, the commonly used definitions of the various states for the consumable item inventory system must be modified.

Definition 2.1: The on-hand inventory (D-B) includes all of the items ready for issue (D) less any backorders (B). Note that the on-hand inventory will be positive if there are no items on backorder.

Definition 2.2: The repair inventory (C) includes all items at the repair facility, including both those undergoing repair and those waiting to begin repair.

Definition 2.3: The net inventory (D-B+C) includes the on-hand inventory and the repair inventory.

Definition 2.4: The inventory position (D-B+C+A) is the sum of the net inventory plus all items which have been placed on order but have not yet been received.

Consider the inventory system as one which has control of a large number of items of a single type where the number of demands (failures) for any time period is a value of a random variable having a Poisson distribution with mean rate λ . The items which fail are assumed to be repairable with probability P and with probability (1-P) the items are scrapped. These failures will be referred to

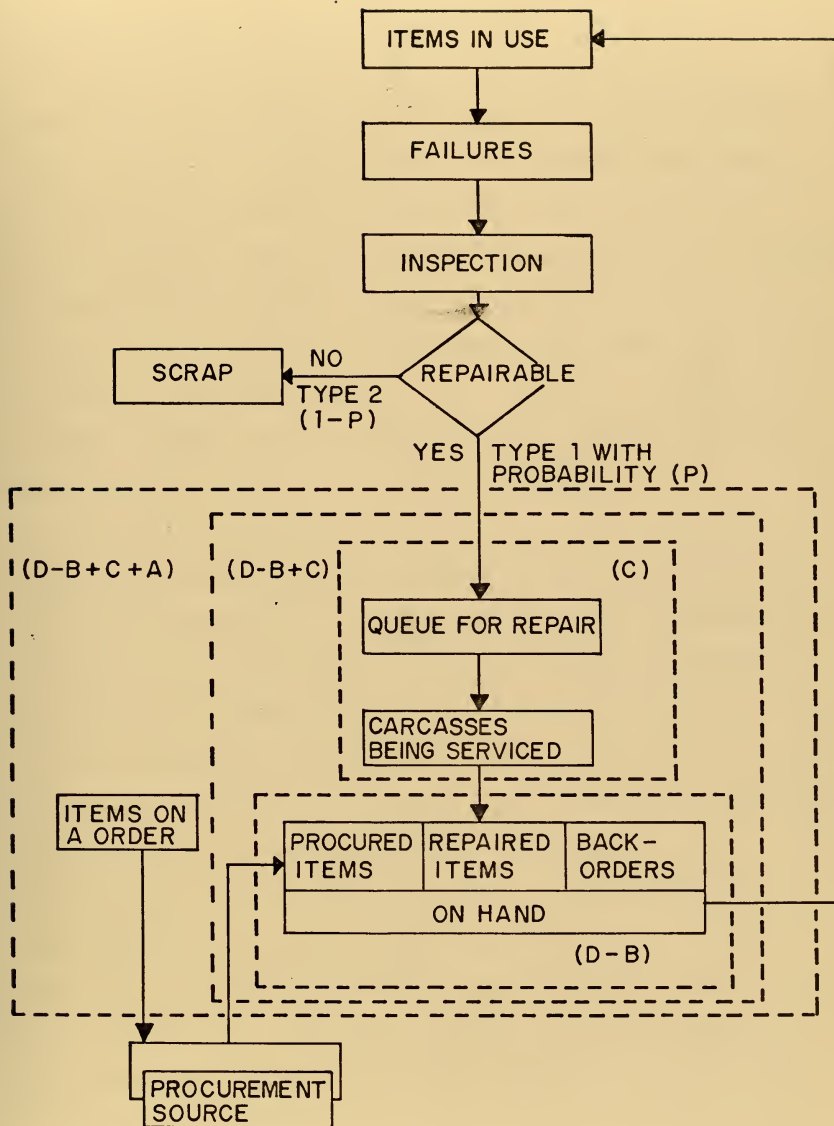


Figure 1. Repairable item inventory system.

as type one and type two, respectively. The manager controlling the system attempts to minimize the time required to fill a demand by maintaining a stock of items ready for issue. Systematically, the manager supplements his stock by procurement of new items and/or receipt of repaired items. Hence the manager is faced with the problems of when and how much to reorder. Standard approaches try to answer these questions by concentrating their efforts on minimizing the total expected cost. However, a difficulty in this approach is the determination of proper weights to assign to the various cost factors involved.

A second approach concentrates on a class of simple procurement policies based on one or two parameters. The goal of this approach is to describe the long-term effect of an arbitrary member of the class of simple procurement policies being considered and then determine the choices for the corresponding parameters which are best with respect to a given measure of performance. This computer simulation allows the user to do this quickly and easily.

After a class of simple policies has been specified, the study of the inventory model becomes a study of the associated stochastic processes and the long run properties of the processes. Emphasis is given to determination of stationary distributions which describe the probabilities of being in the various states after the system has been in operation for a long period of time. This computer program greatly simplifies the procedure whereby actual numbers are produced for these various entities.

The program is capable of representing the repair department in one of two ways. First, as an infinite-server queue with general independent repair times which can be constant or realizations of a uniform, normal, or empirical distribution. Under this policy the repair department acts as a $M/G/\infty$ queueing system which for the random repair times case will allow crossing of repair times. The crossing of orders is also allowed in this case.

The second method considers the repair department as a single server queueing system. That is, only one batch of items up to a maximum quantity (K) can be repaired at one time. Under this policy the user also has the option to enforce a constraint on the minimum batch size (L) as well. This latter policy is provided because frequently in the case of repair, large set-up costs, tool-up costs, or manpower constraints make the second repair policy more economical.

III. EXPERIMENTS

A. OBJECTIVES AND APPROACH

1. Objectives

The objectives of this paper are threefold. (1) To show the need for and justification of such a simulation. (2) To demonstrate the analytical results obtained by Richards [Ref. 7]. (3) To provide a means through which a decision maker can evaluate given or hypothetical inventory systems.

The first objective has been touched on already and will be complemented by the rest of this paper. Consistent with the second and third objectives, a number of experiments were performed which involved generating operating statistics in order to quantitatively compare the simulated results to the theoretical and display the sensitivity of various parameters for particular systems.

2. Approach

Each experiment consisted of a series of simulation runs. Each run simulated up to 14 unit periods, where a unit period represented ten years of actual operation of a given inventory system. For each run, operating statistics were collected for all measures of performance listed in Table I. For illustration purposes two measures of performance were chosen to judge the performance of the hypothetical systems examined in this paper. Those chosen were

TABLE I. SUPPLY PERFORMANCE MEASURES

MOP 1	(F)	= XH100 :	The average number of fills in a unit period.
MOP 2	(FILRT)	= XH99 :	The fill rate multiplied by 100. This is the probability of being able to satisfy a demand without a time delay.
MOP 3	(B)	= XH98 :	The average number of backorders incurred in a unit period.
MOP 4	(B _T)	= XH97 :	The expected number of unit years of backorders in a unit period multiplied by 100.
MOP 5	(BARB)	= XH96 :	The average number of items on backorder at an arbitrary point in time multiplied by 100.
MOP 6	(RDYRT)	= XH95 :	The probability of being able to fill a demand at an arbitrary point in time multiplied by 100.
MOP 7	(ENOH)	= XH94 :	The average number of items on hand at an arbitrary point in time multiplied by 100.
MOP 8	(ORD)	= XH93 :	The average number of orders placed in a unit period.
MOP 9	(RI)	= XH92 :	The average number of repair inductions (batches of items repaired) per unit period. In the infinite server queueing system RI will be C.
MOP 10	(C)	= XH91 :	The average number of items in the repair department at an arbitrary point in time multiplied by 100.
MOP 11	(DIFF 1)	= XH90 :	The difference between ENOH and the theoretical value (V4) of the expected number of items on hand at an arbitrary point in time multiplied by 100.
MOP 12	(DIFF 2)	= XH89 :	The difference between B _T and BARB multiplied by 100. This is calculated to illustrate that the total time-weighted backorders incurred in a unit period is equivalent to the expected number of backorders at an arbitrary point in time.

NOTE: Measure of performance four (B_T) would normally be normalized (\hat{B}_T) by dividing by ten, i.e.,
 $XH97=1102= (\hat{B}_T)=1.102$.

the expected number of unit years of backorders and the average number of items on hand at an arbitrary point in time. For all runs the system was preloaded such that the inventory position was equal to the sum of the order quantity and reorder point. It was felt that this would result in negligible transient times, hence the entire run's statistics were usable in all cases. The input values of all parameters, for what was called the parent system, are summarized in Table II. Deviations from these values of the parent system for any one experiment are pointed out in the description of each experiment as well as in Table III.

TABLE II. PARENT SYSTEM PARAMETER VALUES

PARAMETER	VALUE
Order quantity (Q)	=10
Reorder point (R)	=0,1,2,3...
Minimum batch size (L)	=0
Maximum batch size (K)	=0
Probability of repair (P)	=.9
Mean demand during repair time (\bar{X})	=1.4
Type of distribution for repair time	-constant
Mean repair time (\bar{X}_R)	=336 hours
Lower bound for uniform repair time (LBR)	=0
Upper bound for uniform repair time (UBR)	=0
Variance for repair time distribution (σ_R)	=0

Type of distribution for lead time	-constant
Mean lead time (\bar{X}_L)	=504
Lower bound for uniform lead time (LBL)	=0
Upper bound for uniform lead time (UBR)	=0
Variance for lead time distribution (σ_L)	=0
Type of output desired	-summary

TABLE III. CHANGES TO PARENT SYSTEM PARAMETER VALUES

A. CHANGES FOR EXPERIMENT ONE

(Q)=1,2,3,...,10 (\bar{X})=1.4, 2.75, 4, 5.5

B. CHANGES FOR EXPERIMENT TWO

Type of distribution for repair time--constant,
uniform, normal,
and empirical

Type of distribution for lead time --constant,
uniform, normal,
and empirical

LBR=168 hours LBL=240

UBR=504 hours UBL=768

σ_R = 75 hours σ_L =116

C. CHANGES FOR EXPERIMENT THREE

(Q)=2 (K)=1, 2, 3, 4, 5, 6 (L)=1

D. CHANGES FOR EXPERIMENT FOUR

(Q)=2 (L)=1, 2, 3, 4 (K)=4

E. CHANGES FOR EXPERIMENT FIVE

(Q)=2 (K)=4 (L)=1

(\bar{X})=0.40, 0.65, 0.75, 0.80, 0.90, 1.15, 1.40

B. DESCRIPTION OF EXPERIMENTS

1. First Experiment: The Whole Spectrum

The first experiment considered the parent system but with values of mean demand during repair time of 1.4, 2.75, 4, and 5.5. Also, order quantities of one through ten and variable reorder points were allowed. That is, for each combination of demand and order quantity above, the reorder point (R) was allowed to vary from an initial value of zero until the expected number of unit years of backorders reached an acceptable stationary level. This was done in order to see how the measures of performance, particularly the two mentioned above, varied with respect to changes in the order quantity and reorder point for the system described above.

The amount of computer time required to run one period ranged from a low value of 5.89 seconds for $(\bar{X})=1.4$, with $Q=10$ and $R=7$; to a high value of 28.57 seconds for $(\bar{X})=5.5$, with $Q=1$, and $R=0$. The total computer time required for this experiment would obviously depend on the particular inventory system being investigated by the user. It is assumed that the user would know, determine, or estimate his system demand such that he could restrict his investigation to one or two values. Therefore, the number of runs and total computer time required for his experiments would be significantly less than that required for these presented in this paper. Some typical results of the first experiment are shown in Figures 2 and 3.

FIGURE 2

TYPICAL RESULTS OF EXPERIMENT ONE
 REORDER POINT (R) VS UNIT YRS. OF BACKORDERS (B_T)
 WHERE: $\bar{X} = 5.5$ PER UNIT PERIOD

Q = 1 ◀
 Q = 3 ▼
 Q = 6 ▶
 Q = 9 ▲

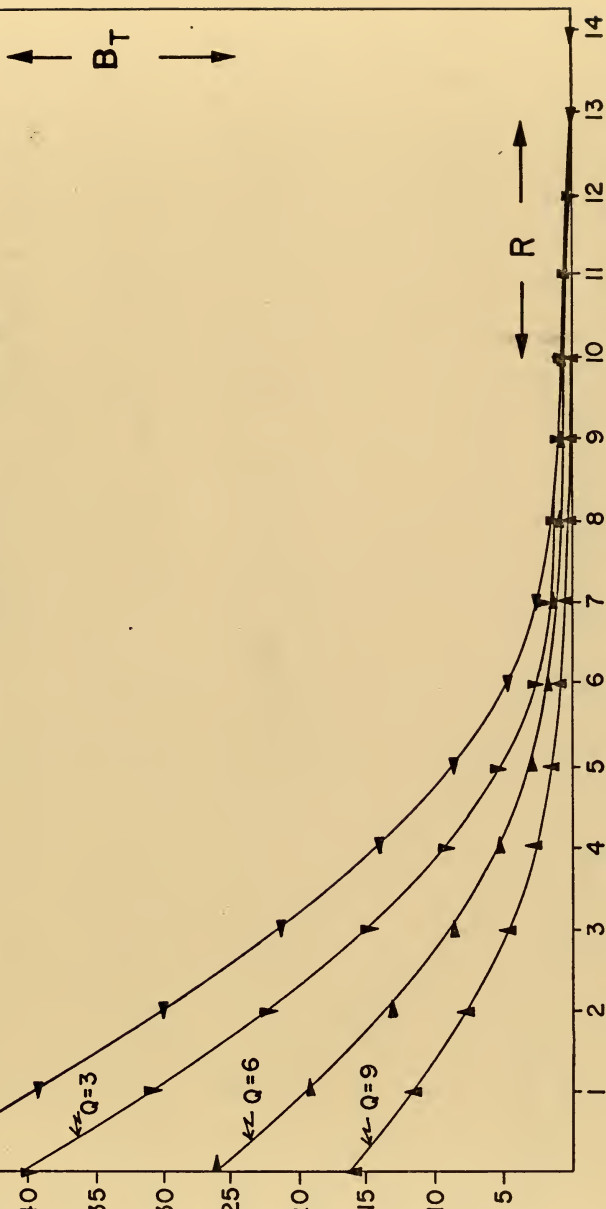


FIGURE 3

TYPICAL RESULTS OF EXPERIMENT ONE

REORDER POINT (R) VS. UNIT YRS. OF BACKORDERS (B_T)
PER UNIT PERIOD

WHERE:

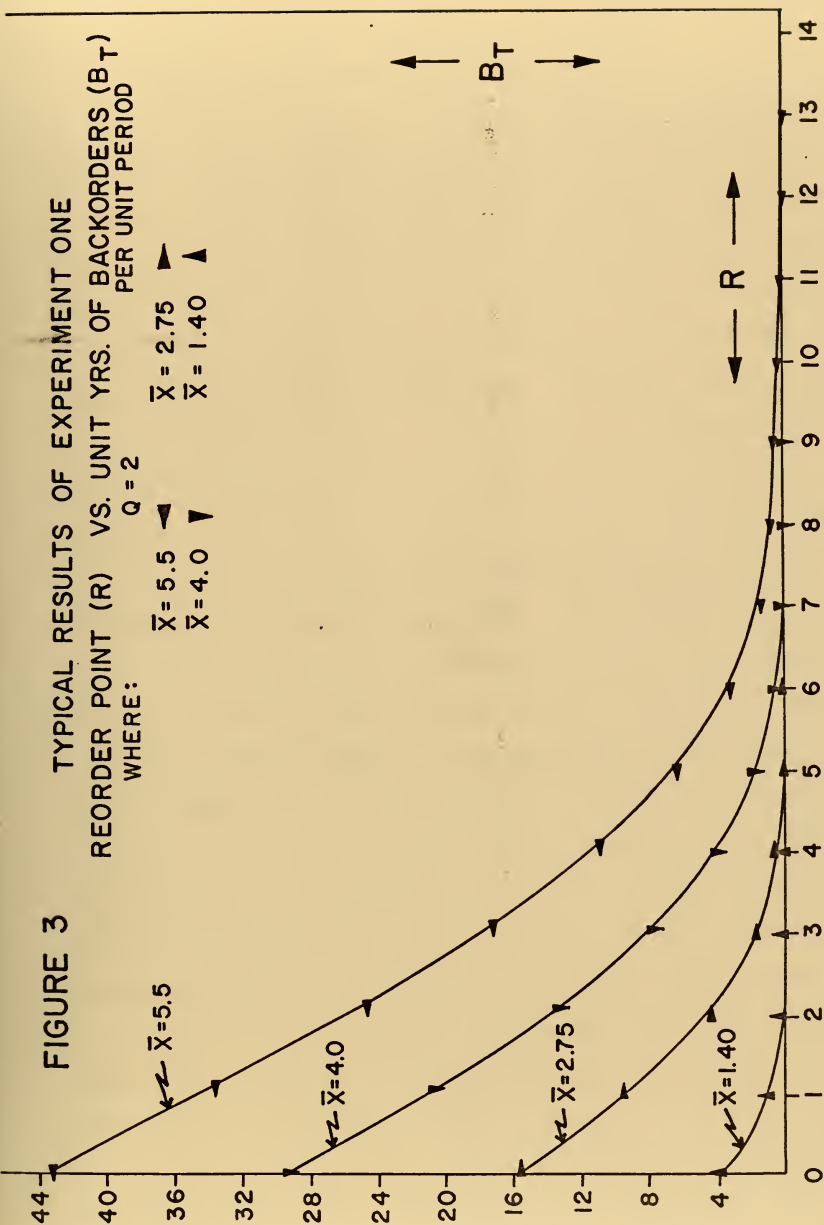
$Q = 2$

$\bar{X} = 5.5$ ▲

$\bar{X} = 2.75$ ▲

$\bar{X} = 4.0$ ▼

$\bar{X} = 1.40$ ▲



2. Second Experiment: Distribution Test

For this experiment a representative system from experiment one was chosen for further study. The system chosen was thought to be a typical, low demand, high cost, repairable item inventory system with a reorder quantity of ten and a mean demand during repair time of 5.5. The particular reorder quantity used is a somewhat subjective choice which the manager would make. Hopefully the manager could look at the results of experiment one and pick what would be the best quantity. A procedure to do this is explained in the analysis of experiment one. However, what will often happen is that one of several possible factors will force the manager to live with some predetermined order level. Some examples of these may be: there is a minimum number of items that the manager can let a contract for with a particular manufacturer, or an engineering requirement forces items to be procured in matched pairs or sets, or possibly a fixed unit of issue by the manufacturer. Regardless of how it is decided upon, once the reorder quantity is fixed the sensitivity of the system can be investigated by changing various input factors such as the repair time distribution or lead time distribution, as was done in this experiment. Constant, uniform, normal, and empirical repair and lead time distributions were examined. For each combination of distributions the reorder point (R) was varied until the total unit years of backorders was stationary. The purpose of this experiment was to show the sensitivity or

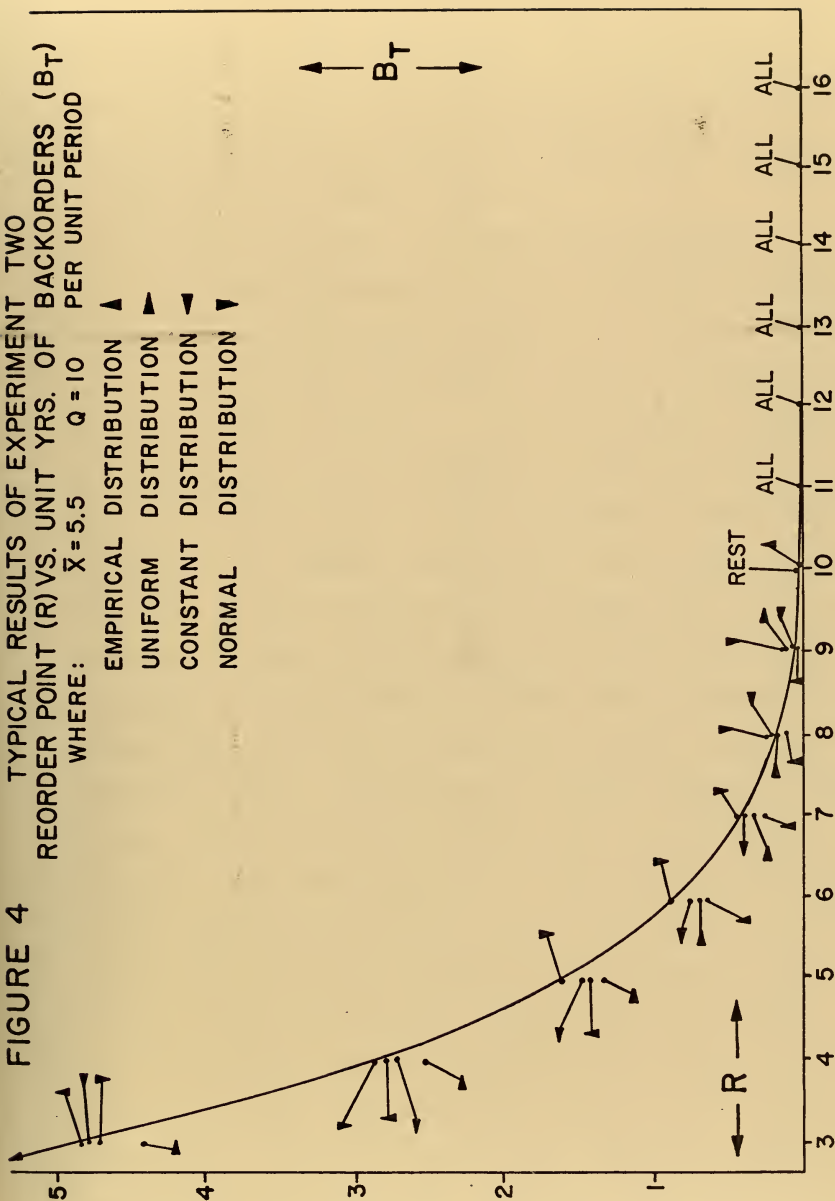
FIGURE 4

TYPICAL RESULTS OF EXPERIMENT TWO

REORDER POINT (R) VS. UNIT YRS. OF BACKORDERS (B_T)

WHERE: $\bar{X} = 5.5$ $Q = 10$ PER UNIT PERIOD

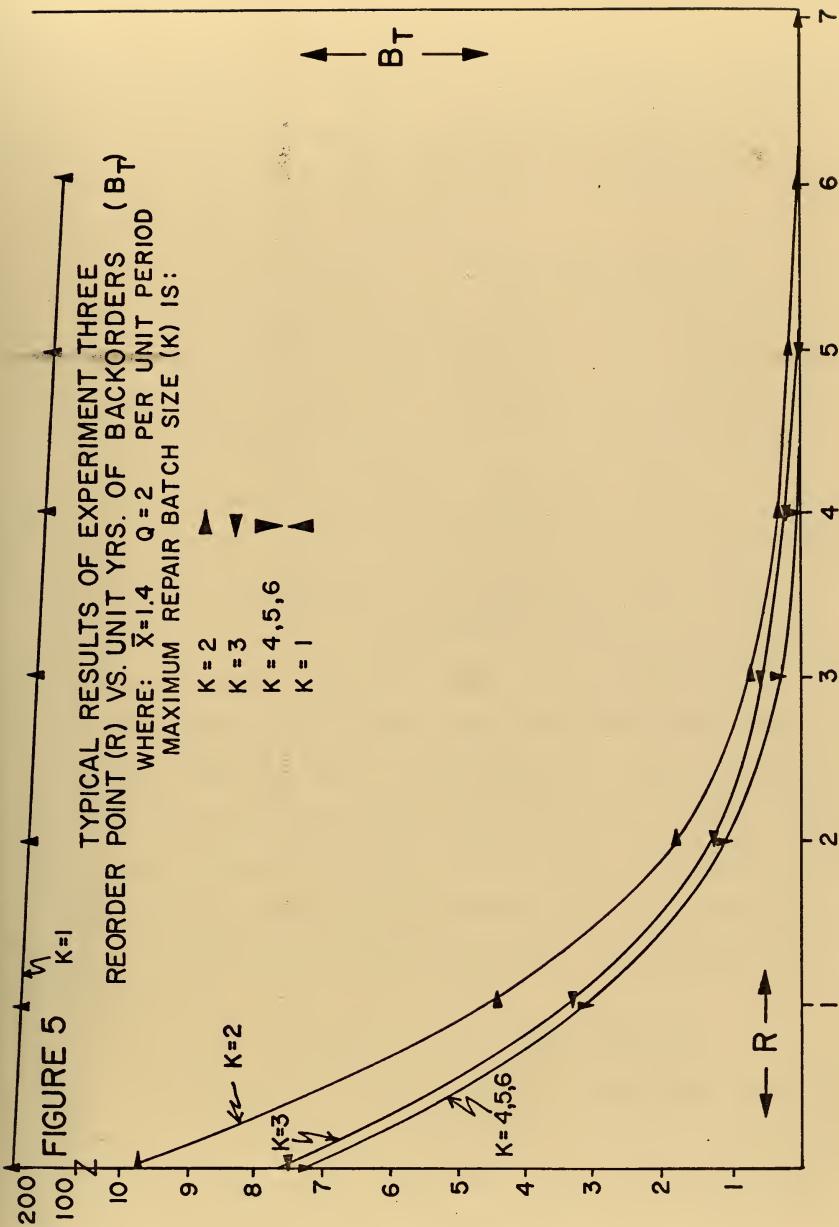
EMPIRICAL DISTRIBUTION ▲
 UNIFORM DISTRIBUTION ▴
 CONSTANT DISTRIBUTION ▼
 NORMAL DISTRIBUTION ▾



insensitivity of the modified parent system to various types of distributions for the repair and lead times each having the same mean and variance. Some typical results of the experiment are given in Figure 4.

3. Third experiment: Maximum Batch Size Test

For this experiment the parent system was modified in order to treat the repair department as a single server queueing system where the server is capable of handling as many as K items at one time. Using the procedure explained in the analysis of experiment one, the order quantity was fixed at a value of two for this experiment. Then the sensitivity of the reorder point (R) was investigated by varying R for each feasible value of K , the maximum batch size, until the expected number of unit years of backorders reached an acceptable stationary level. A simple way to determine the maximum value for K is to look at the results obtained in experiment one for the comparable infinite server queueing system. The run in experiment one where all the input parameter values were identical to this one with the exception of the parameters associated with the repair discipline would be the comparable infinite server queueing system. One of the items included in the summary output for Storage Statistics, FASTR, summarizes the statistics for the case in which the repair department is treated as an infinite server queue. Two of the particular statistics included there are the average number of items in the repair department and the maximum number of items that were ever in the



repair department at any time. These particular values were used in this experiment as a guide for determining the maximum value for K to be considered. The results of this experiment are shown in Figure 5.

4. Fourth experiment: Minimum Batch Size Test

In this experiment the parent system was again modified to treat the repair department as a single server queueing system. Again, as in the last experiment, the sensitivity of the reorder point to the type of repair policy used was investigated. This time, however, K rather than L was held constant while R was varied for each value of L, the minimum batch size, until the expected number of unit years of backorders reached an acceptable stationary level. The values of L were varied from one to K. The value of K used was four, which was the maximum feasible quantity for K as determined by experiment three. Since the repair department was treated as a single server queueing system, the repair of one batch of items must be completed before repairs can begin on the next batch of items. The results of this experiment are presented in Figure 6.

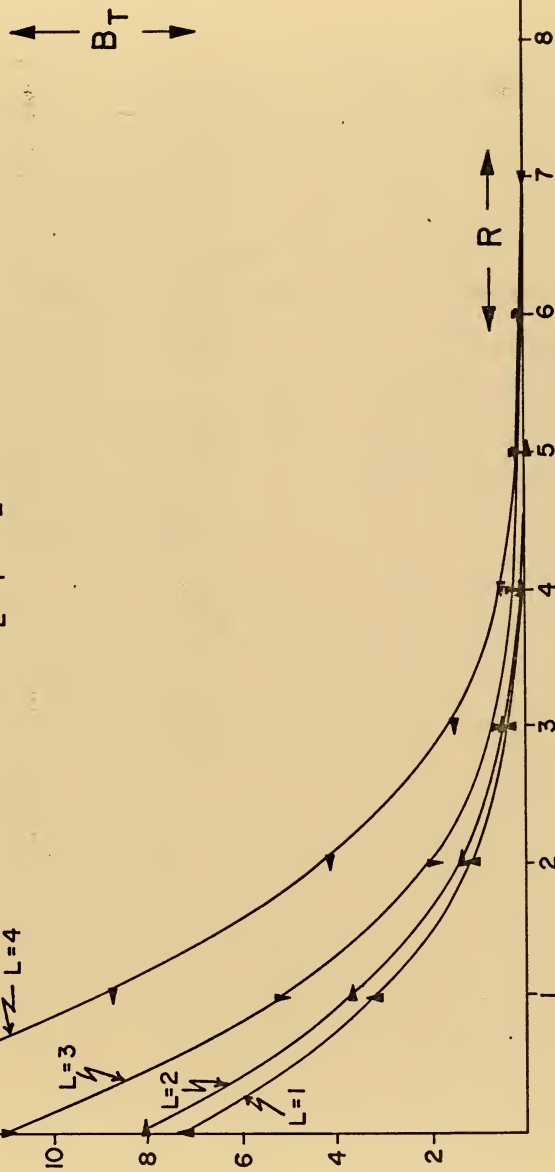
5. Fifth Experiment: Mean Repair Time Test

The modified parent system of experiment four was used for this experiment but with $L=1$ and with various mean demands during repair time $\overline{(X)}$. The mean demand during repair time was allowed to vary from 1.40 to 0.40 by changing the mean time to repair from 336 hours to 96 hours

FIGURE 6

TYPICAL RESULTS OF EXPERIMENT FOUR
 REORDER POINT (R) VS. UNIT YRS. OF BACKORDERS (B_T)
 WHERE: $\bar{X} = 1.4$ $Q = 2$ PER UNIT PERIOD
 MINIMUM REPAIR BATCH SIZE (L) IS:

$L = 4$ ◀
 $L = 3$ ▼
 $L = 2$ ▶
 $L = 1$ ▲



while keeping the mean time between failures constant at 242 hours. The sensitivity of the reorder point was then investigated by again varying R for each value of \overline{X} until the expected number of unit years of backorders per unit period reached a stationary level. A physical interpretation here could be a change of equipment (perhaps manual repair to automation) or a change in the budget for the repair department. An investigation of the results of this experiment suggested a comparison of the trade off between decreasing the mean repair time (\overline{X}_R) and increasing the maximum batch size in the repair department. The results of experiment five are presented in Figure 7.

336 HR

FIGURE 7

TYPICAL RESULTS OF EXPERIMENT FIVE

REORDER POINT (R) VS. UNIT YRS. OF BACKORDERS (B_T)

WHERE:

 $X = 1.4$ $Q = 2$

PER UNIT PERIOD

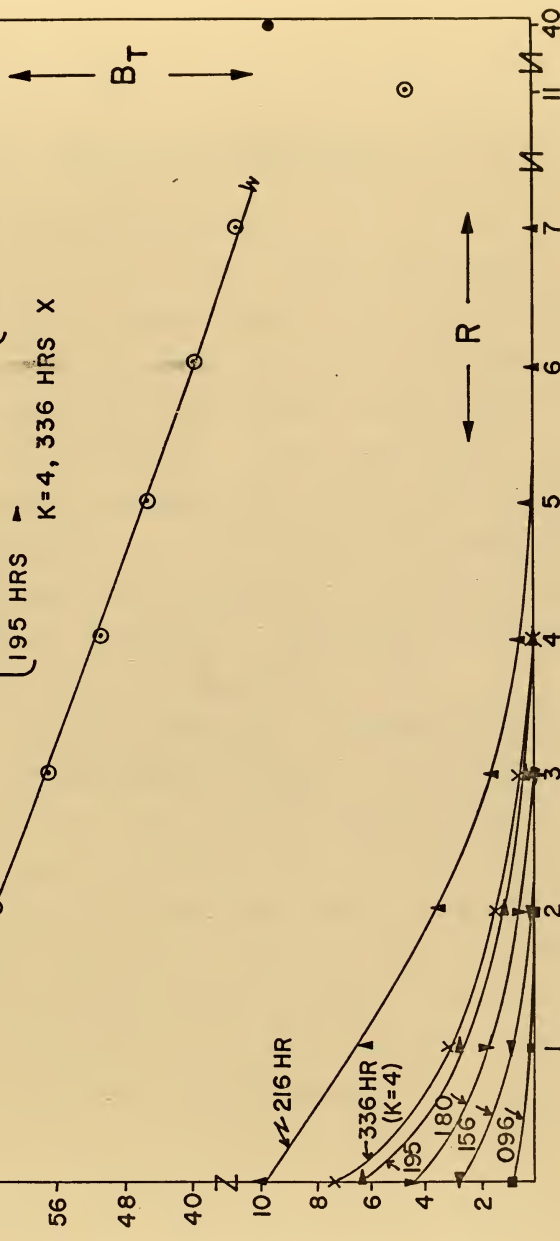
MEAN TIME TO REPAIR ONE BATCH OF ITEMS:

$K=1$

{ 336 HRS ●
 276 HRS ○
 216 HRS ▲
 195 HRS ▴

$K=1$

{ 180 HRS ▼
 156 HRS ◀
 096 HRS ■

 $K=4, 336 \text{ HRS } X$ 

IV. ANALYSIS AND CONCLUSIONS

A. ANALYSIS OF THE FIRST EXPERIMENT

In all experiments the reorder quantity and reorder level were controlled. In addition, for each experiment one variable such as the maximum or minimum repair batch size was controlled while the remaining variables were determined from the simulation. In this experiment, for each reorder quantity and mean demand during repair time considered, the reorder level which reduced the expected number of unit years of backorders per unit period to an acceptable level was determined. To determine this reorder point a one-way analysis of variance was used.

Such a test requires additivity, linearity, normality, independence, and homogeneous variances. The first two assumptions were felt to be satisfied since the expected number of unit years of backorders per unit period appears to be a linear sum of the true mean effect, the true effect of the treatment (the value of the reorder point chosen), and the true effect of the errors observed. The primary error in the observations is due to the stream of random numbers. It seems to be a good assumption that these errors are normally distributed with mean zero and variances which are equal for each treatment.

This particular test was used because it is quite robust, that is, the test is reliable under rather strong modifications

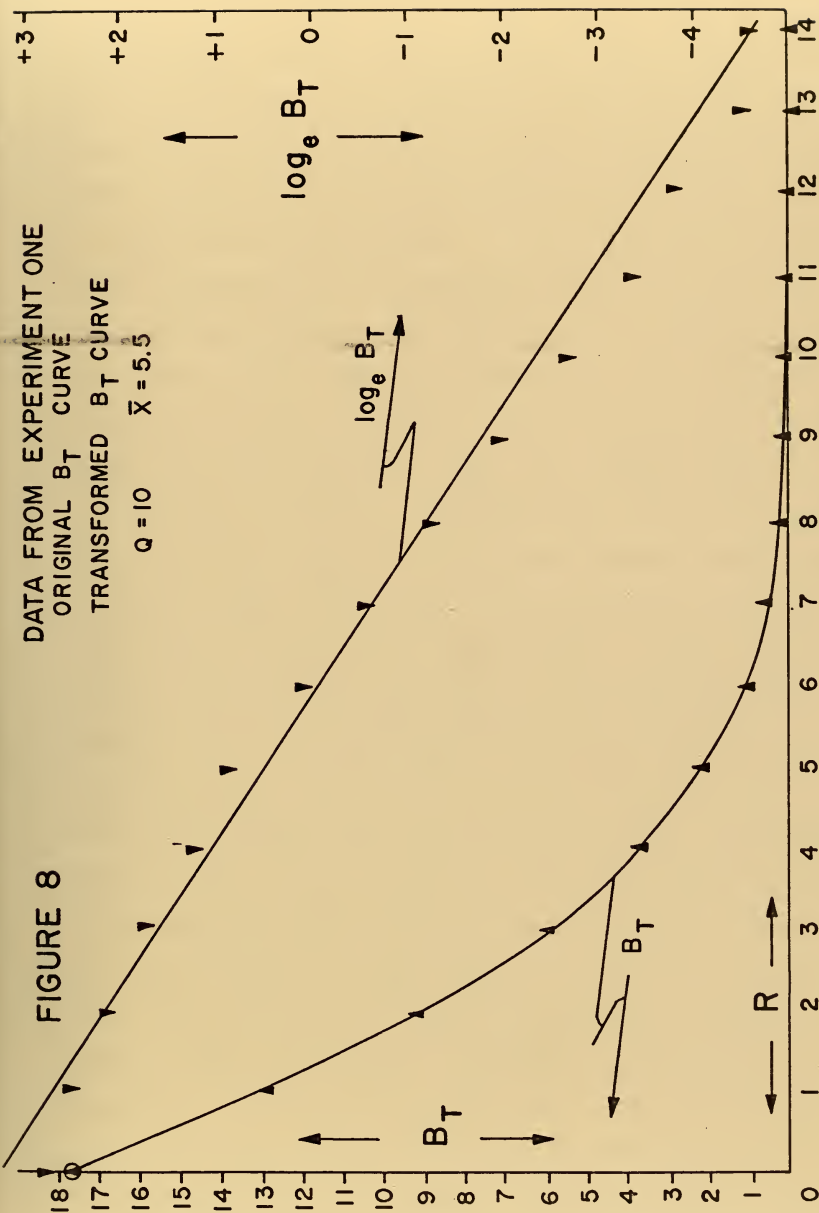
to the above assumptions. It can also be shown that transformations of the original data are justified. This was particularly important in this case because it was necessary to make a logarithmic transformation to the observations in order to obtain a linear model. Figure 8 illustrates this type of transformation.

The null hypothesis used in this analysis of variance was that the expected number of unit years of backorders per unit period for each of the reorder points was the same for a fixed reorder quantity and mean demand during repair time. For the parameters considered in this paper the analysis of variance was reported for that case which appeared to have the highest probability of not rejecting the null hypothesis. Nine observations with different streams of random numbers for each run were made and the null hypothesis was tested at the .01 significance level. The results of the analysis of variance yielded a test statistic of 194, well in excess of the F-value (2.48) required to reject the null hypothesis. (For calculations see page 79.)

Since the primary goal of the first experiment was to determine the "best" order quantity, reorder point combination, Scheffe's [Ref. 5] multiple comparison test was then used to find this combination. This procedure tests for equality of means between contrasts in a systematic fashion until a significant difference between means is found. The analysis of variance indicated that a significant difference

FIGURE 8

DATA FROM EXPERIMENT ONE
 ORIGINAL B_T CURVE
 TRANSFORMED B_T CURVE
 $Q = 10$ $\bar{X} = 5.5$



was present. Scheffe's test was then used to determine the minimum value of the difference in means which statistically differentiated among the various values of the expected number of unit years of backorder per unit period.

In order to apply Scheffe's test a (Q,R) pair yielding a value of the measure of performance which was considered acceptable was chosen. The first contrast looked at the differences in the measure of performance for this pair (Q,R_M) and the pair (Q,R_{M-1}) . If this contrast failed to reject the null hypothesis, that the expected number of unit years of backorders per unit period were equal at the .01 significance level, then the next contrast would examine the pairs (Q,R_M) and (Q,R_{M-2}) . This procedure continued until the null hypothesis was rejected. If this happened when the pairs (Q,R_M) and (Q,R_{M-j}) were compared, the minimum value of the differences which was statistically significant was known to be less than or equal to the difference between the values of the measure of performance for these two pairs. For future reference denote R_{M-j} by R_Q . The minimum difference required for significance can be solved for exactly by applying this procedure in reverse, that is, by setting the value of an arbitrary contrast equal to the test statistic required to reject the null hypothesis and then transforming this contrast into the difference required in the measure of performance for significance. For the example systems considered in this paper this procedure required the use of three contrasts

before significance was found between (Q,R) pairs (10,11) and (10,8). This contrast resulted in a test statistic of 6.99 which was greater than the F-value (2.48). Hence, the null hypothesis was rejected. (For calculations see pages 81-83). Further, forcing the test statistic to be equal to 2.48 and solving for the required minimum difference between values of the measure of performance for significance resulted in a value of 0.096 unit years of backorders per unit period. Statistically, one cannot distinguish a difference between 0.096 and 0.00 for this measure of performance. For this particular example, which is summarized in Tables IV and V, this means that any position to the right of the dashed line in Table IV could be an acceptable operating point for a system when X , the mean demand during repair time, was 5.5 and the other system characteristics were as noted in Tables II and III.

The procedure then used to determine the "best" operating point for this particular example was to list the pairs (Q, R_Q) and observe the measures of performance listed in Table I. Notice from Table V that when one tries to identify the pair that minimizes both the average number of items on hand at an arbitrary point in time and the average number of procurements per unit period the (Q, R_Q) pairs (2,10) and (3,10) are selected as the "best" operating points for this example. A simple comparison of holding cost and procurement cost could enable the user to further distinguish between these two choices.

TABLE IV. TYPICAL RESULTS FROM EXPERIMENT ONE WITH B_T SIGNIFICANCE LINE
 DRAWN IN SUMMARY TABLE OF THE EXPECTED NUMBER OF UNIT YEARS OF BACKORDERS
 PER UNIT PERIOD FOR VARIOUS (Q,R) PAIRS

$\bar{X}=5.5$ (mean demand during repair time)

Order Quantity Q	0	1	2	3	4	5	6	7	8	9	10	11	12
1	49.20	39.42	30.08	21.35	14.19	8.62	4.71	2.30	0.97	0.34	0.10	0.05	0.02
2	43.38	33.86	24.95	17.22	10.93	6.45	3.38	1.52	0.56	0.20	0.09	0.05	0.02
3	39.88	30.63	22.09	14.84	9.09	5.00	2.39	1.02	0.40	0.16	0.08	0.05	0.02
4	35.18	26.13	18.56	12.42	7.64	4.23	2.12	0.96	0.40	0.18	0.09	0.05	0.02
5	29.90	21.99	15.40	10.13	6.22	3.44	1.78	0.80	0.30	0.13	0.06	0.03	0.02
6	26.08	19.02	13.07	8.50	5.21	2.92	1.48	0.69	0.30	0.14	0.08	0.05	0.02
7	24.54	17.73	12.17	7.78	4.56	2.45	1.29	0.64	0.25	0.08	0.03	0.00	0.00
8	20.33	14.30	9.60	6.07	3.57	1.94	0.94	0.43	0.18	0.09	0.03	0.01	0.00
9	16.04	11.17	7.50	4.70	2.78	1.33	0.59	0.22	0.08	0.02	0.00	0.00	0.00
10	17.61	12.93	9.07	6.07	3.74	2.17	1.17	0.60	0.20	0.13	0.04	0.01	0.00

TABLE V. SUPPLY PERFORMANCE MEASURES FOR EXPERIMENT ONE
SUMMARY STATISTICS FOR, RIGHT ADJACENT, (Q,R) PAIRS ADJACENT TO B_T SIGNIFICANCE LINE
 $\bar{X}=5.5$ (MEAN DEMAND DURING REPAIR TIME)

Order Quantity Q	FILLS	X% FILRT	B	B _T	BARB	X% RDYRT	OH	ORD	RI	C	DIFF 1	DIFF 2	R _Q
10	1471	99.50	6	0.06	0.00	100	9.52	15	1337	5.08	-0.20	.006	10
9	1464	99.12	13	0.08	0.01	99.38	7.61	16	1337	5.08	0.39	.007	8
8	1464	99.12	13	0.09	0.00	100	7.89	18	1337	5.08	0.37	.009	9
7	1457	98.65	20	0.08	0.00	99.38	7.12	21	1337	5.08	-0.10	.008	9
6	1465	99.19	12	0.08	0.00	100	7.73	24	1337	5.08	0.34	.008	10
5	1470	99.43	7	0.06	0.00	100	7.20	29	1337	5.08	-0.01	.006	10
4	1462	98.99	15	0.09	0.00	100	6.77	36	1337	5.08	0.04	.009	10
3	1465	99.19	12	0.08	0.00	100	6.14	47	1337	5.08	-0.08	.008	10
2	1462	98.99	15	0.09	0.00	100	5.76	71	1337	5.08	0.04	.009	10
1	1470	99.43	7	0.05	0.00	100	6.17	141	1337	5.08	-0.05	.005	11

In applying these results to other systems it was observed that the value of 0.096 for the expected number of unit years of backorders per unit period could also be used to determine the (Q, R_Q) candidates for the "best" operating point when considering all values of mean demand during repair time less than or equal to 5.5. It was further observed that among these (Q, R_Q) candidates, most measures of performance would differ by negligible amounts with the exception of the average number of items on hand at an arbitrary point in time and the average number of procurements made per unit period.

B. ANALYSIS OF EXPERIMENTS TWO THROUGH FIVE

1. General Comments

The analysis of variance and the multiple comparison test could have been used to obtain candidates for optimal operating points with other relevant parameters, such as the minimum or maximum repair batch sizes or the distribution of repair time being used as the basis. Although this analysis was not repeated for all experiments, a close observation of the data seemed to indicate that the minimum significant difference value would be nearly the same regardless which parameter was chosen for the basis. Use of this observation was made in the analysis of experiments two through five.

2. Repair Time Distribution (Second Experiment)

The results summarized in Tables VI and VII showed that the type of repair time distribution had a negligible

TABLE VI. TYPICAL RESULTS FROM EXPERIMENT TWO WITH B_T SIGNIFICANCE LINE DRAWN IN.
SUMMARY TABLE OF THE EXPECTED NUMBER OF UNIT YEARS OF BACKORDERS PER UNIT PERIOD

FOR VARIOUS (DIST'N TYPE, R) PAIRS

$\bar{X}=5.5$ (MEAN DEMAND DURING REPAIR TIME) $Q=10$ (REORDER QUANTITY)

DISTRIBUTION TYPE	0	1	2	3	4	5	6	7	8	9	10	11	12
CONSTANT	15.29	10.91	7.49	4.76	2.75	1.46	0.75	0.41	0.22	0.11	0.06	0.03	0.00
UNIFORM	15.39	10.62	7.07	4.40	2.55	1.32	0.69	0.37	0.21	0.12	0.06	0.03	0.01
NORMAL	15.10	10.95	7.42	4.73	2.87	1.66	0.89	0.46	0.25	0.13	0.05	0.01	0.00
EMPIRICAL	15.69	10.84	7.46	4.81	2.82	1.45	0.65	0.26	0.10	0.03	0.01	0.00	0.00

TABLE VII. SUPPLY PERFORMANCE MEASURES FOR EXPERIMENT TWO

SUMMARY STATISTICS FOR, RIGHT ADJACENT, (DIST'N TYPE, R) PAIRS ADJACENT TO B_T SIGNIFICANCE LINE

DISTRIBUTION TYPE	FILLS	% FILRT	B	B_T	BARB	% RDYRT	OH	ORD	RI	C	DIFF 1	DIFF 2	R
CONSTANT	1421	99.29	10	.06	.005	99.40	9.61	14	1292	5.05	-0.10	.001	10
UNIFORM	1418	99.08	13	.06	.005	98.81	9.63	14	1292	5.02	-0.08	.001	10
NORMAL	1417	99.01	14	.05	.011	98.27	9.58	14	1292	5.08	-0.13	-.006	10
EMPIRICAL	1423	99.35	8	.03	.000	100	8.69	14	1292	4.96	-0.02	.073	9

effect on the expected number of unit years of backorders per unit period provided the mean repair times were the same. This result was expected since Palm's theorem indicates that the distribution of the number of items in repair depends only on the repair time distribution through its mean value. When the probability of repair was nearly one it was also observed that the type of lead time distribution had a negligible effect on system performance. A probable explanation for this result is that the average number of orders placed per unit period was insignificant compared to the number of repairs made. This can be seen in Table VII.

3. Maximum Batch Size Test (Third Experiment)

In this experiment the effects of change in the maximum repair batch size were investigated. The results summarized in Tables VIII and IX indicate a vast improvement in system performance for increasing the maximum batch size from one to two but negligible improvement thereafter. The initial improvement can be explained by the queueing result which says that in order to ensure stationarity the maximum repair batch size must be large enough to guarantee that the mean time between failures multiplied by K must be greater than the mean repair time. An important question which was considered was exactly how large K should be. In order to provide an answer further work would have to be done to determine an explicit rule for finding the "best" value of K . However, it is felt that this rule would depend on the utilization factor of the repair department. For this

TABLE VIII. TYPICAL RESULTS FROM EXPERIMENT THREE WITH B_T SIGNIFICANCE LINE DRAWN IN. SUMMARY TABLE OF THE EXPECTED NUMBER OF UNIT YEARS OF

BACKORDERS PER UNIT PERIOD FOR VARIOUS (K,R) PAIRS

MAX BATCH SIZE (K)	$\bar{X}=1.4$ (MEAN DEMAND DURING REPAIR TIME)									
	<div><div></div><div>R, REORDER POINTS</div></div>									
	0	1	2	3	4	5	6	7	...	40
1	227.20	193.90	185.90	177.70	170.00	162.40	154.80	146.50	...	31.70
2	9.68	4.35	1.78	0.57	0.23	0.07	0.01	0.00		
3	7.49	3.31	1.24	0.41	0.16	0.03	0.01	0.00		
4	7.26	3.18	1.19	0.37	0.13	0.03	0.01	0.00		
5	7.20	3.13	1.15	0.33	0.13	0.03	0.01	0.00		
6	7.20	3.13	1.15	0.33	0.13	0.03	0.01	0.00		

TABLE IX. SUPPLY PERFORMANCE MEASURES FOR EXPERIMENT THREE

SUMMARY STATISTICS FOR, RIGHT ADJACENT, (K,R) PAIRS ADJACENT TO B_T SIGNIFICANCE LINE

	FILLS	FILRT	B	B_T	BARB	RDYRT	OH	ORD	RI	C	DIFF 1	DIFF 2	R
2	339	99.12	3	.07	.006	96.95	4.24	17	218	2.01	-0.80	.001	5
3	339	99.12	3	.03	0	96.95	4.39	17	213	1.86	-0.65	.003	5
4	339	99.12	3	.03	0	97.65	4.44	17	213	1.81	-0.60	.003	5
5	339	99.12	3	.03	0	97.65	4.45	17	212	1.80	-0.59	.003	5
6	339	99.12	3	.03	0	97.65	4.45	17	212	1.46	-0.59	.003	5

particular example, increasing K to obtain a utilization factor of less than .417 resulted in negligible improvement in system performance.

4. Minimum Batch Size Test (Fourth Experiment)

This experiment considered the effects of change in the minimum batch size for a fixed value of K. The results summarized in Tables X and XI indicated that, as before, changes in the minimum batch size requirement had little effect in overall system performance with the exception of the average number of repair inductions. If the cost of a repair induction is large certain economies could be achieved by requiring a minimum batch size. This could be achieved without degrading system performance. Further investigation would be needed to generalize these results in order to obtain a rule for determining the "best" value for the minimum batch size.

5. Mean Repair Time Test (Fifth Experiment)

For the case in which both K and L are one, the effects of change in the mean repair time were examined. The results summarized in Tables XII and XIII indicated that, as expected, the magnitude of the mean repair time is highly significant. In particular, system effectiveness was inversely proportional to the mean repair time.

TABLE X. TYPICAL RESULTS FROM EXPERIMENT FOUR WITH B_T SIGNIFICANCE LINE DRAWN IN.

SUMMARY TABLE OF THE EXPECTED NUMBER OF UNIT YEARS OF BACKORDERS PER UNIT PERIOD

FOR VARIOUS (L,R) PAIRS

 $\bar{X}=1.4$ (MEAN DEMAND DURING REPAIR TIME)

K=4

Q=2

MIN BATCH SIZE (L)	R, REORDER POINTS							
	0	1	2	3	4	5	6	7 8
4	15.65	8.88	4.17	1.57	0.50	0.13	0.02	0.01 0
3	11.06	5.23	1.97	0.69	0.23	0.08	0.02	0 0
2	7.97	3.59	1.24	0.40	0.13	0.02	0	0 0
1	7.26	3.18	1.19	0.37	0.13	0.03	0.01	0 0

TABLE XI. SUPPLY PERFORMANCE MEASURES FOR EXPERIMENT FOUR

SUMMARY STATISTICS FOR, RIGHT ADJACENT, (L,R) PAIRS ADJACENT TO B_T SIGNIFICANCE LINE

MIN BATCH SIZE	FILLS	FILRT	B	B_T	BARB	RDYRT	OH	ORD	RI	C	DIFF 1	DIFF 2	R
4	339	99.12	3	0.02	0	97.56	4.60	17	77	2.67	-0.73	.003	6
3	337	98.67	5	0.08	0	97.56	3.84	17	101	2.41	-1.20	.008	5
2	340	99.41	2	0.02	0	98.17	4.26	17	142	1.99	-0.78	.002	5
1	339	99.12	3	0.03	0	97.56	4.44	17	213	1.81	-0.60	.003	5

TABLE XII. TYPICAL RESULTS FROM EXPERIMENT FIVE WITH B_T SIGNIFICANCE LINE DRAWN IN.

SUMMARY TABLE OF THE EXPECTED NUMBER OF UNIT YEARS OF BACKORDERS PER UNIT PERIOD

FOR VARIOUS (TIME, R) PAIRS

 $\bar{X}=1.4$, MEAN TIME BETWEEN DEMANDS=242 HOURS, $Q=2$, $K=1$, $L=1$

MEAN TIME TO REPAIR IN HOURS		R, REORDER POINTS						
		0	1	2	3	4	5	6
336	227.20	193.90	185.90	177.70	170.00	162.40	154.90	146.50
276	79.20	70.70	63.30	57.10	51.30	45.70	40.30	35.00
216	10.16	6.16	3.41	1.62	0.56	0.16	0.01	0
195	6.20	2.97	1.08	0.23	0.04	0	0	0
180	4.68	1.90	0.44	0.03	0	0	0	0
156	2.83	0.84	0.08	0	0	0	0	0
096	0.81	0.11	0	0	0	0	0	0
*336 (K=4)	7.26	3.18	1.19	0.37	0.13	0.03	0.01	0

*This entry is included here so one can compare decreasing mean time versus increased K.

TABLE XIII. SUPPLY PERFORMANCE MEASURES FOR EXPERIMENT FIVE

SUMMARY STATISTICS FOR, RIGHT ADJACENT, (TIME, R) PAIRS ADJACENT TO B_T SIGNIFICANCE LINE

MEAN REPAIR TIME (HOURS)	Q=2		K=1															
	FILLS	FILRT	B	B _T	BARB	RDYRT	OH	ORD	RI	C	DIFF 1	DIFF 2	R					
336 (K=1)	329	96.19	13	31.70	3.33	96.95	19.26	17	260	24.80	--	--	40					
276	327	95.62	15	0.59	0.06	92.89	10.86	17	312	11.44	--	--	21					
216	340	99.41	2	0.01	0	99.40	5.59	17	315	1.80	-0.89	.001	6					
195	336	98.25	6	0.04	0	98.81	4.02	17	315	1.37	-0.54	.004	4					
180	335	97.96	7	0.03	0	98.22	3.27	17	315	1.19	-0.35	.003	3					
156	326	95.92	14	0.08	0	93.49	2.54	17	316	0.85	-0.17	.008	2					
096	342	100.00	0	0	0	99.40	2.94	17	318	0.46	0	0	2					
*336 (K=4)	339	99.12	3	0.03	0	97.65	4.44	17	213	1.81	-0.60	.003	5					

*This entry is included here so one can compare decreasing mean time versus increased K.

C. CONCLUDING COMMENTS ABOUT THE ANALYSIS

It is interesting to note the tradeoffs between various parameters. For instance, it seemed that one could determine a reduction in mean repair time which would give an equivalent increase in system performance to that provided by an increase in the maximum repair batch size. An illustration of such a tradeoff for the example considered in this paper can be seen by looking at Table XIII for the following entries: minimum and maximum batch size of one with mean repair time of 216 versus minimum batch size of one, maximum batch size of four, and mean repair time of 336 hours.

Decisions such as determining the maximum batch size or mean repair time must be made by the decision maker based on the feasibility of change and the cost of such a change. This simulation will provide the manager with each of the measures of performance needed to make such decisions.

In addition to the above remarks, another item should be discussed. Recall that one of the objectives of this study was to demonstrate the analytical results obtained by Richards [Ref. 7]. Notice that in each of the tables which summarized the measures of performance for the various experiments, two of the measures of performance listed are "DIFF 1" and "DIFF 2." "DIFF 1" reflects the difference between the simulated and theoretical values for the average number of items on hand at an arbitrary point in time. "DIFF 2" reflects the difference between the expected number

of unit years of backorders per unit period and the average number of items on backorder at an arbitrary point in time. In observing these differences in Table V it was noted that the maximum value for "DIFF 1" was 0.39 and for "DIFF 2" it was .009. This represents a maximum percentage difference of 5.1% for "DIFF 1." It is felt that since these differences are so small, this simulation does in fact demonstrate the validity of the analytical results obtained by Richards.

V. PROGRAM GUIDE

This section is written so that, along with Table I (Supply Performance Measures) and the table of symbols and abbreviations, it can be a self contained unit for the user. It contains all the information needed to successfully set up and run the model.

A. GENERAL COMMENTS

Most people will agree that any inventory system should be designed to give the maximum amount of service for the minimum amount of cost possible. Hence most inventory studies evaluate the various policies being considered by looking at the costs incurred over a unit period of time. Then policy A is chosen over policy B if the total expected cost per unit time using policy A is less than when using policy B. However, quite often in this approach one cannot specify all the cost elements. An alternative approach is

to determine the average values of such measures of performance as the number of items on hand at an arbitrary point in time, the number of backorders per unit period, and others as listed in Table I. The user can then choose from these measures of performance those he wishes to use to evaluate the policies under his scrutiny. This method provides the user with an easy way of deciding which values for system parameters such as the reorder quantity and reorder point yield satisfactory performance.

Closed form solutions can be obtained for the various measures of performance but often the solutions are not very tractable for further calculations. This is where the simulation is of value. It systematically and quickly calculates the magnitude of all the measures of performance for various input values of all parameters involved and presents the user with a fast and easy means of investigating the sensitivity of his inventory system to any changes in these parameters. To try and do this procedure by hand or apply the analytical results to a sensitivity analysis of this size would involve a large number of man-hours of work to produce effective results.

B. EVENTS

This model is an event-step computer simulation written in the GPSS/360 language, i.e., all actions that are to occur in the simulation are dynamically generated by the computer program as a result of previous simulation actions

and are listed chronologically in order of descending priority in a Current Events Chain, Future Events Chain, or Interrupt Chain. For purposes of programming, each of the major actions included in the simulation can be considered as an entity in itself and assumes the form of a computer program subroutine. Such major actions have been called events for convenience. The eight major actions (events) included in the simulation are:

- (1) Item Failure
- (2) Item Inspection
- (3) Scrap
- (4) Repair
- (5) Issue
- (6) Backorder
- (7) Order
- (8) Receipt of Material

Each of the computer program subroutines representing these events uses as input parameters the following information:

- (1) Time the event is to occur
- (2) Identification of event
- (3) Number of items involved
- (4) Priority level (0,1,...)
- (5) Current values of system characteristics such as inventory position (IP), number of items ready for issue (D), and others.

The dynamic process of simulating one inventory policy from start to finish forms the executive routine for the computer simulation. This executive routine also includes three program subroutines referred to as Random Observation, Calculation of Measures of Performance, and Incrementation of System Parameters. General flow charts describing the logic included in each event of the simulation plus the



interrelationship of events are included as Figures 9 through 12. Detailed flow charts for the entire model are provided in Appendix B. Random observation is the subroutine that generates random points in time at which certain parameter values are recorded in order to obtain certain useful statistics about the system. Calculation of Measures of Performance is the subroutine that, at the completion of any unit period (a time period which represents 10 years of actual system operations), interrogates the information contained in all system parameters and transfers control of the computer program to the calculation phase of the measures of performance listed in Table I. Incrementation of system parameters is the subroutine which increments the current value of R, the reorder point, by one. It also clears the results of the last unit period from the program and resets all the random number generators to the same values used in the previous run to ensure meaningful comparisons between runs.

C. INPUT

All inputs to the computer program are contained on three punched cards called Initial Cards and four Function Follower Cards. The format for this information and the units to be used are detailed in Tables XIV through XVI. With one set of input cards a user can simulate the interactions involved for as long as 140 years or 14 unit periods of ten years each. Each unit period will increment the

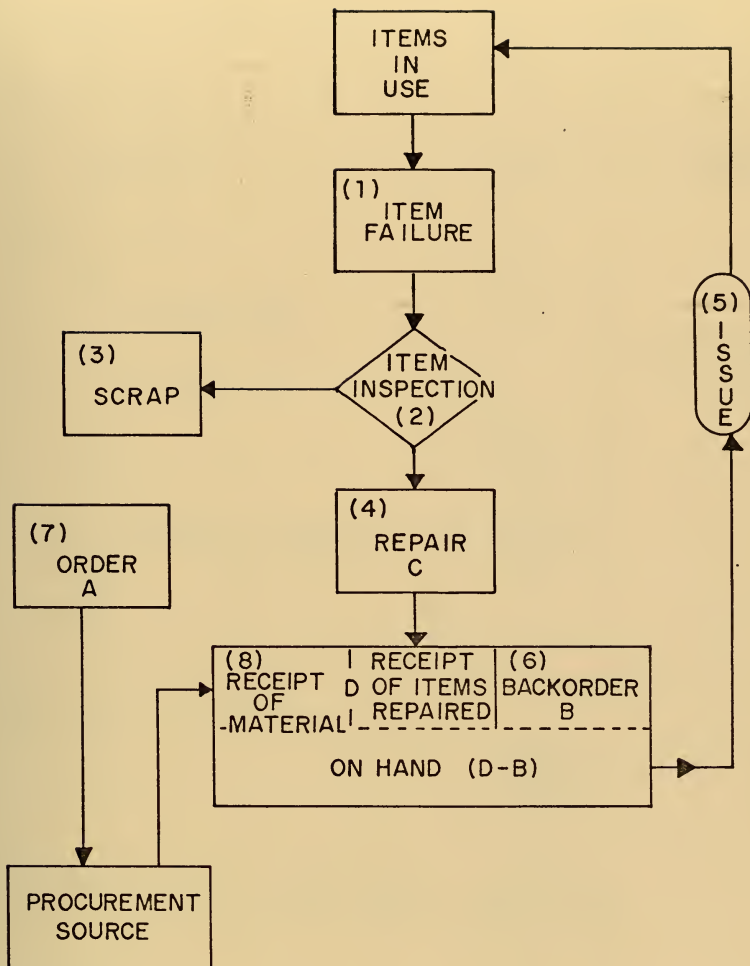


Figure 9. Relationships of events.

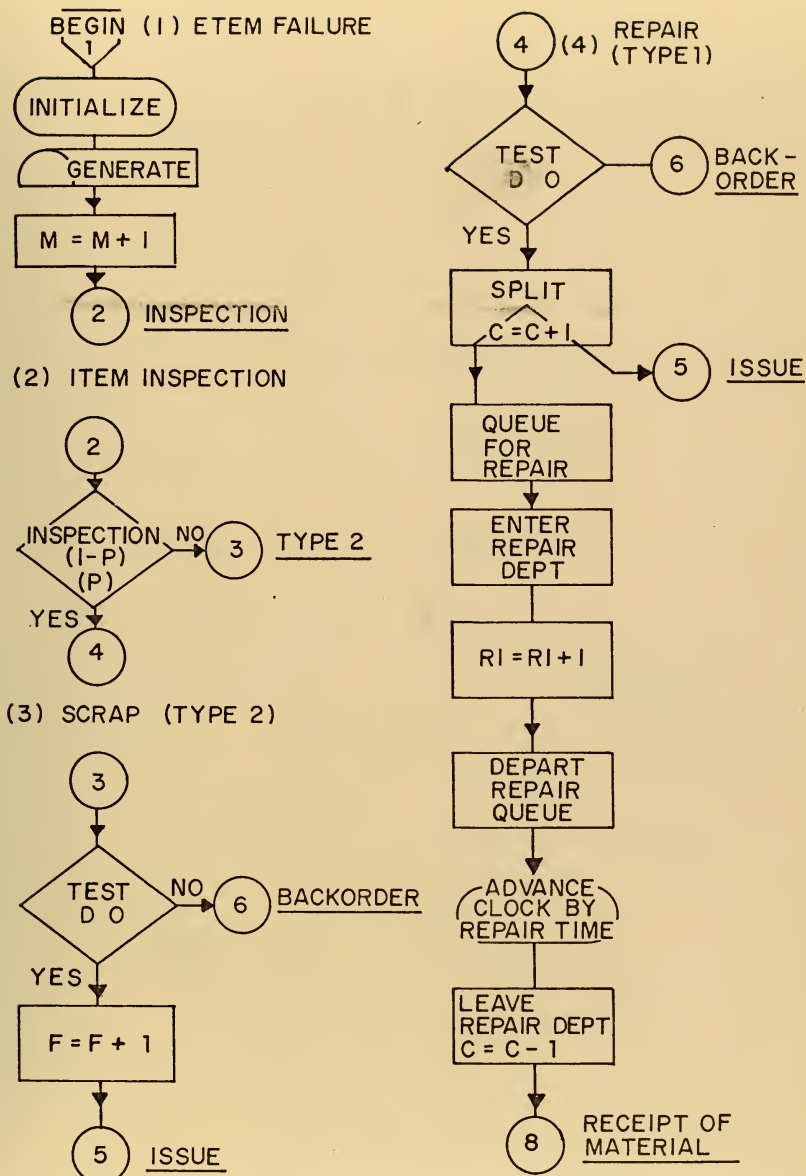


Figure 10. Major Events (1)-(4) Logic Diagrams.

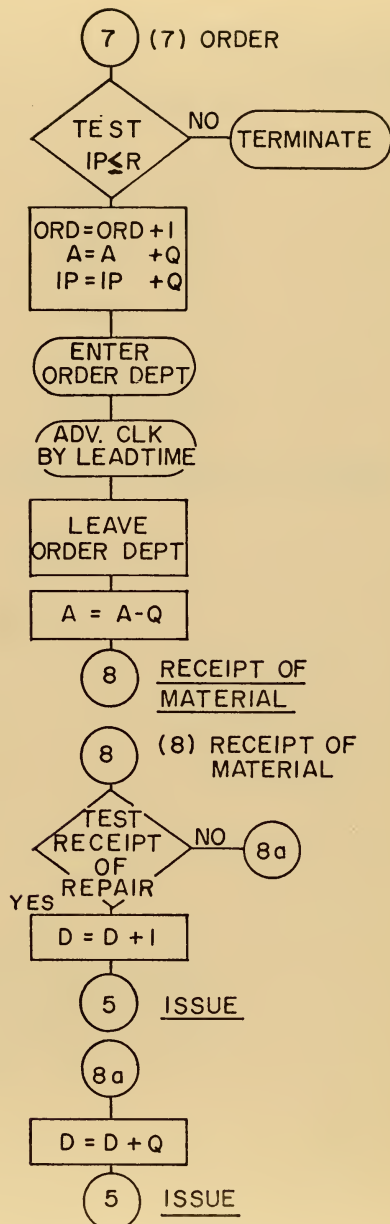
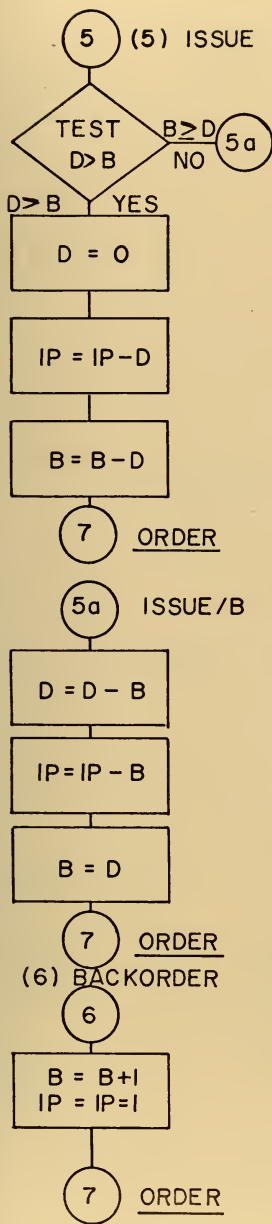
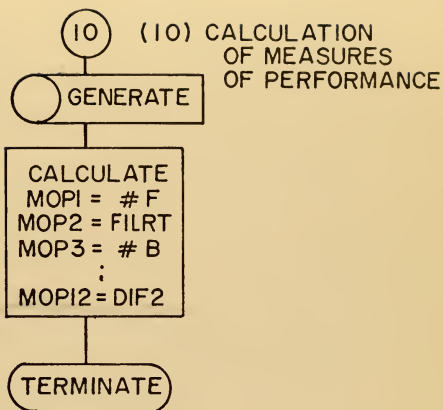
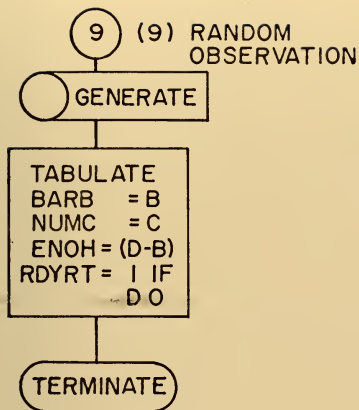
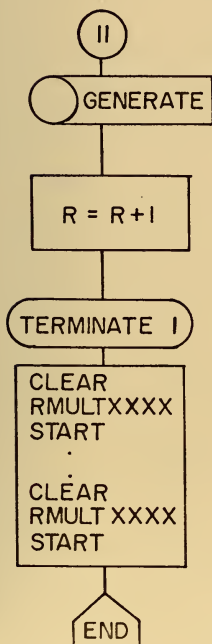


Figure II. Major Events (5)--(8) Logic Diagrams.



(II) Incrementation of system parameters and control of program running time.



This clears all values from program. It also resets the random number generator so the same number of XA's are created for each successive unit period.

Figure 12.
Administrative events (9) – (II) logic diagrams.

value of R by one starting with the initial value chosen by the user (generally zero). In order for the user to vary parameter values other than the reorder level, a new set of initial cards must be prepared by the user for the program. All input values must be integers and in units of hours if it is a time input. For example in the three columns allotted for Q on card one the user could enter anything from 001 to 999 for one item up to 999 items or in the columns allotted for μ_0 , the mean time between demands (failures), on card two the user could enter anything from 999 to 001 to represent failures rates of .7308 per month (one every 999 hours) to 730 per month (one every hour).

D. FORMAT SECTION

This section contains the tables which identify and explain the various system parameters and characteristics needed as user input in order to use the simulation. It also includes the feasible range for each entity. This input must be typed by the user on three Initial Cards and four Function Follower Cards.

1. Description Of Entities Found On The Initial Cards

This input is summarized in Tables XIV through XVI.

2. Description Of Function Follower Cards One and Two

These cards contain the function values, Y_i , and argument values, X_j , for the empirical distribution of repair time. This is a discrete numerical valued function with up to ten different points. Figure 12 shows that this

TABLE XIV. FIRST INITIAL CARD

Q	Reorder quantity. Number of items reordered at one time.
R	Reorder Point. If Inventory Position is $\leq R$ an order is placed.
L	Minimum Batch Size. Under the batch repair policy, repair won't start unless at least L units are currently waiting to be repaired. Ensure L=0 if the infinite server queueing policy for repair is desired.
K	Maximum Batch Size. Under the batch repair policy, this is the maximum number of items that can be repaired at one time.
P	Probability of Repair. The probability that a failed item can be repaired. If P=0000, (P=0.0) the model degenerates to the consumables case. If P=1000, (P=1.0) the model becomes one without degradation of supply which implies that no orders are placed and no outside procurement is needed.
μ_D	Mean time between failures. The expected number of hours between demand.
FLAG/R	Repair Time Distribution Flag. One of four numbers.
3	Constant repair time.
4	Uniform distribution for repair time.
5	Normal distribution for repair time.
6	Empirical distribution for repair time.

TABLE XV. SECOND INITIAL CARD

μ_R	Mean Time For Repair. The expected time required for repair of an item or batch of items.
LBR	Minimum Repair Time. The lower bound on repair time for the uniform distribution of repair time (used when FLAG/R=4).
UBR	Maximum Repair Time. The upper bound on repair time for the uniform distribution of repair time, (used when FLAG/R=4).
σ_R	Standard Deviation For Repair Time. The standard deviation for the normal distribution of repair time, (used when FLAG/R=5).
FLAG/R	Lead Time Distribution Flag. One of four numbers. 7 Constant lead time. 8 Uniform Distribution for lead time. 9 Normal Distribution for lead time. 10 Empirical Distribution for lead time.
μ_L	Mean Time For Procurement. The expected value for procurement lead time.

TABLE XVI. THIRD INITIAL CARD

LBL	Minimum Lead Time. The lower bound on lead time for the uniform distribution of lead time.
UBL	Maximum Lead Time. The upper bound on lead time for the uniform distribution of lead time.
σ_L	Standard Deviation For Lead Time. The standard deviation for the normal distribution of lead time.
XH17	Type Of Output Desired Flag. One of two numbers. Zero for summary output, or one for standard output. (Normally a zero should be used to conserve printing time.)

function has the same function value, $FN_j = Y_i$, for all argument values in the interval between X_{j-1} and X_j . No interpolation is performed, and the value at the right-hand end of the interval is used. For argument values less than the magnitude of X_1 , $FN_1 = Y_1$. For argument values greater than the last value of X_j , $FN_n = Y_n$.

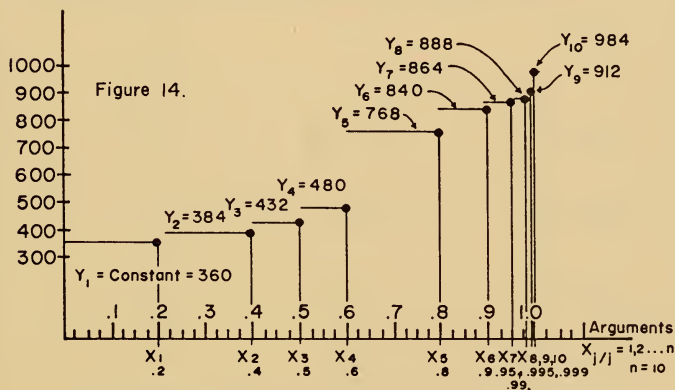
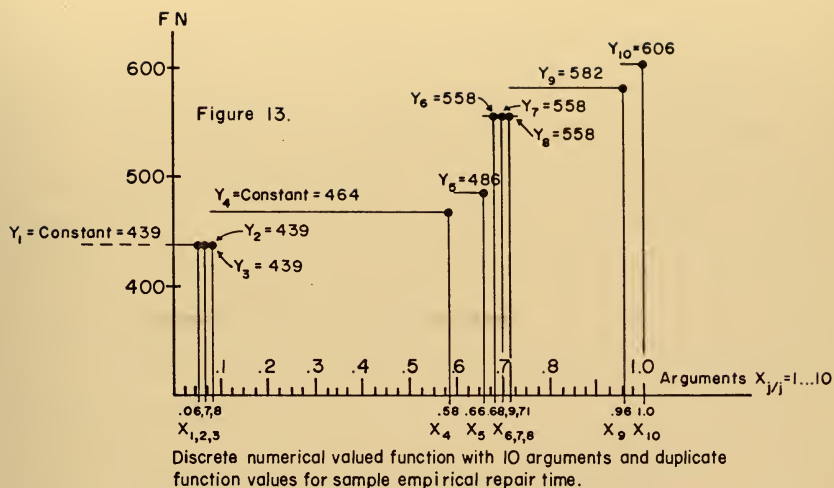
The function follower cards contain the individual points (successive pairs of X_j , Y_j values) of the function. For example, consider the function pictured in Figure 12 as the repair time distribution, the function follower cards would then be: 0.06,439/0.07,439/0.08,439/0.58,464/0.66,486/0.68,558/0.69,558/0.71,558/0.96,582/1.0,606.

Notice that in this example there are really only six different function values FN_j , so argument values .06, .07, .68, and .69 are dummy values and are put in only to meet the requirement of having ten pairs of X_j, Y_j values.

3. Description Of Function Follower Cards Three and Four

These cards are used exactly the same way as Function Follower Cards One and Two except they contain the function and argument values for the empirical distribution of lead time. For example, if the function pictured in Figure 13 were the lead time distribution, FFC's three and four would be: 0.2,360/0.4,384/0.5,432/0.6,480/0.8,768/0.9,840/0.95,864/0.99,888/0.995,912/0.999,984.

Notice that once FLAG/R and FLAG/L have been selected the program ignores those cards which involve the parameters of the other distributions. Therefore only those



values or cards which affect the parameter values of the particular type of distribution specified need be worried about. For example, if FLAG/R=5 and FLAG/L=10, the user must ensure μ_R , σ_R and μ_L have the values desired and FCC's three and four have the values desired. The values of LBR, UBR, LBL, UBL, σ_L , and FCC's one and two can be ignored.

4. Examples of Initial Cards

Tables XVII through XIX give examples of the three Initial Cards.

TABLE XVII. EXAMPLE OF INITIAL CARD ONE

Columns	Name	Range	Units
8-14	INITIAL		
19-22 23-24	XH1; Q	1-99	ITEMS
25-29 30-31	/XH2, R	0-99	ITEMS
32-36 37-38	/XH3, L	0-K	ITEMS
39-43 44-45	/XH4, K	0-99	ITEMS
46-50 51-54	/XH5, P	0-1000	.001
55-59 60-62	/XH6, μ_D	0-999	HOURS
63-67 68-69	/XH7, FLAG/R	3,4,5, or 6	NONE

All input information is right-justified in the space allotted. Example:

COLUMN NUMBER -	28	39	55	69
	↑	↑	↑	↑
INITIAL	XH1,10/XH2,01/XH3,00/XH4,01/XH5,0900/XH6,061/XH7,03			

TABLE XVIII. EXAMPLE OF INITIAL CARD TWO

Columns	Name	Range	Units
8-14	INITIAL		
19-22 23-25	XH8, μ_R	0-999	HOURS
26-30 31-33	/XH9, LBR	0-998	HOURS
34-39 40-43	/XH10, UBR	0-998	HOURS
44-49 50-52	/XH11, σ_R	0-580	HOURS
53-58 59-60	/XH12, FLAG/L	7,8,9, or 10	NONE
61-66 67-69	/XH13, μ_L	0-999	HOURS

All input information is right-justified in the space allotted. Example:

COLUMN NUMBER - 34 53 69
 ↑ ↑ ↑
INITIAL XH8,336/SH9,168/XH10,0504/XH11,095/XH12,07/XH13,504

TABLE XIX. EXAMPLE OF INITIAL CARD THREE

Columns	Name	Range	Units
8-14	INITIAL		
19-23 24-26	XH14, LBL	0-998	HOURS
27-32 33-36	/XH15, UBL	0-1998	HOURS
37-42 43-45	/XH16, σ_L	0-580	HOURS
46-51 52	/XH17,	0, or 1	NONE

All input information is right-justified in the space allotted. Example:

COLUMN NUMBER - 26 37 51
 ↑ ↑ ↑
INITIAL XH14,240/XH15,0768/XH16,116/XH17,0

E. OUTPUT

The computer program has two different sets of output, summary (special) and standard. The type of output received depends on the value of XH17. If XH17=0 the summary output is received, but if XH17=1 the standard output is received. Computer running and printing time is greatly increased if the standard output is used. Therefore, it is recommended that XH17 be set to zero. If there is a major error in the program, the standard output will be printed out automatically. Following is an explanation of the two sets of output.

1. Summary Output

An example of the summary output is illustrated in Appendix D. Notice that there are three major divisions: (1) savevalues, (2) storage statistics and (3) queue statistics.

a. Savevalues

There are four subdivisions in this part. The first is a list of nonzero halfword savevalues between five and 100 for system parameters and statistics. This list is included for completeness only and not because of their nature or importance. The second subdivision, Clock Time, merely shows the number of hours of operation simulated. Normally this will be 87600 hours or ten years of operation, which has been defined to be a unit period. The third subdivision, Critical System Parameters, lists the values of Q, K, and L which the user has chosen and input on the initial cards. It also prints out the value

of R for this particular time period. The last subdivision, Measures of Performance, includes most of the important statistics to be considered when judging or comparing alternative inventory systems.

b. Storage Statistics

This division provides the important statistics for the order department (ORDER), the infinite server queueing repair process (FASTR), and the single server queueing repair process (SLOWR). The important items and explanation of each storage follow. (1) Storage -- the name used to represent the physical entity. (2) Capacity -- the maximum number of transactions which can use the storage at any one time. (3) Average Contents -- the average number of transactions that were in the storage during the run. (4) Entries -- the number of transactions which entered the storage. For SLOWR this is the number of batches of items that entered the repair process rather than the number that were repaired as in the FASTR storage. Likewise, it is the number of orders placed rather than the number of items ordered for the ORDER storage. (5) Average Time/Tran -- the average number of hours that an order was outstanding, or the average number of hours that it took for an item to be repaired, or the average number of hours it took for a batch of items to be repaired. (6) Maximum Contents -- the maximum number of transactions that used the storage at any time during this run.

c. Queue Statistics

This division reflects the important statistics for the infinite server queue repair department (FSTRQ) and the single server queue repair department (SLOWQ). The important items and explanation of each follow. (1) Queue -- the name used to represent the physical entity. (2) The explanation for maximum contents, average contents, and average time/tran are the same as for storages. (3) Total Entries -- the number of transactions which entered the queue. (4) Zero Entries -- the number of transactions which entered the queue and departed with no time delay. Note that for the FSTRQ it should always be the same as the total entries. (5) Average Time/Trans -- the average number of hours that the delayed transactions, only, spent waiting to be repaired.

2. Standard Output

An example of the standard output is illustrated in Appendix D. Notice that the standard output includes all of the items listed in the summary output as well as the following items. (1) table statistics, (2) group counts, and (3) block counts.

The table statistics provide the statistics of the following tables: BTCHZ, BTIME, ENOH, NBARB, NUMC, and RDYRT. The important items and explanations of each follow. (1) Table -- the name used to collect the particular statistic. (2) Entries in Table -- the number of times that an entry was made in the table. (3) Mean Argument --

the average value of the entries made in the table.

(4) Standard Deviation -- the standard deviation of the entries made in the table. (5) Upper Limit -- the maximum value of each cell of the table. For example, all entries between 51 and 100 fall in cell two of BTIME table.

(6) Observed Frequency -- the number of entries which fall into this cell. (7) Percentage of Total and Cumulative

Percentage are self-explanatory. (8) Multiple of Mean --

the upper limit of the cell expressed as a multiple of the mean of the table. The additional pieces of information

given in the tables as well as the queue and storage

statistics which are not explained in detail here are those which, under normal circumstances, are of no concern to the

user. However, there may be occasions when a user wishes

to make changes internal to the program such as adding the

capability of handling compound Poisson demands. In such

a case the standard output would be a necessity, especially

the block counts, in helping the user debug the program.

The reason this output was included is to show the user

that this additional information is available from the

computer, and to show the user the type of output he will

receive by default should an error be made while using the

model. If the user desires to receive the standard output,

XH17 should be set to one on initial card three. If one

desires further information relating to computer output,

he should see the IBM User's Manual [Ref. 4].

VI. USES AND APPLICATIONS OF THE MODEL

This model was built for three major reasons. First and foremost was to give a convenient and useful tool to present day inventory managers faced with the decisions of how much to buy, when to buy, and how to conduct repairs for the items under their cognizance. As was pointed out earlier, there are presently two major approaches in vogue to help the manager solve his problem; cost analysis and the use of simple procurement policies with specified measures of supply performance. Under the cost analysis approach the user runs into the difficult problems of quantifying costs. For instance I, the inventory holding cost, consists of many things, some of which all managers agree on, such as storage and handling, and some which they don't agree on, such as loss, breakage, and opportunity cost. Similar disagreements concern the fixed order cost as well. For example, the Navy uses a sliding scale from \$25 to \$75, while the Army uses a different sliding scale from \$50 to \$400. Further problems arise when one tries to assign a cost to a critical item being out of stock when needed. Primarily because of these difficulties in quantifying costs, the second approach was used in this paper. It was felt that statements such as "a fill rate of 95% must be maintained" would at least mean the same thing to all managers as well as being something that all managers could agree on.

The second reason this model was built was to provide an efficient way of obtaining insight into the problems faced by the inventory manager. Simulation is a straightforward and logical way of doing this. In addition simulation avoids some of the pitfalls of the more classical means of analysis. For example, the model can use any time-step desired and simulate as far into the future as desired. The model easily handled the different types of repair processes which analytically prove to be quite cumbersome. When these reasons are coupled with the combined effects of uncertainty, dynamic interaction of system events, and interdependence among system parameters, it is easily seen why this problem was particularly amenable to simulation. Richards [Ref. 7] showed that analytical expressions could be obtained for the individual measures of performance used in this paper. However, to actually generate numbers from these expressions for a sensitivity analysis of this size would be a monumental task. Therefore, simulation again seemed to be called for. Also, the advantage of time compression, both in being able to simulate into the future in a matter of seconds and being able to obtain numerical values for the various measures of performance, made simulation appealing.

A major criticism of simulation is that the simulated queueing model yields only an estimate of parameters, such as a waiting line's average length or associated probability of delay, and as such is subject to statistical

error. A second criticism is that if the system is so complicated and complex that the other methods fail and a simulation is required, it will necessarily be a very difficult job to build the model and analyze its results.

In this particular case the first disadvantage acted to a much lesser degree since analytical expressions for all the measures of performance used in this model do exist. This fact allowed less replication to obtain trustworthy answers in the search for the best choices of the policy parameters. A moderate number of runs in a reasonable period of time accompanied with the statistical tests of section four also lent credibility to the answers and conclusions obtained. The second disadvantage noted went for naught since the model exists and works.

Finally, the third reason that this model was constructed was to demonstrate the validity of certain analytical expressions for some of the measures of performance used in this paper. As was pointed out in section four, the model succeeded in doing this. From the comments above and the results presented in this paper, it is easily seen that the model can be used effectively to describe current inventory systems, to explore hypothetical inventory systems, and to design or improve given inventory systems, as well as be used as a teaching aid. The simulation model can be used to help answer such a current question as how to best conduct repairs on the engines for the new class of destroyers DD963.

In applying this model to any of the uses mentioned above, one is always faced with the problem of deciding which measure of performance to use as the criterion for making decisions. In this paper the expected number of unit years of backorders per unit period was used. One reason it was used over the average number of items on backorder per unit period was that it considers both the number of backorders and the amount of time the backorders were outstanding, while the other merely considers the number of backorders, totally ignoring the amount of time the backorders were outstanding. It was felt that both the fill rate and the ready rate were even worse than the average number of backorders since they don't even take into consideration the number of units backordered, but merely note whether or not any backorders exist. Perhaps the most significant reason the expected number of unit years of backorders per unit period was used as the criterion was that once it was considered acceptable, the remaining measures of performance were also acceptable.

There were occasions when another measure of performance, the average number of items on hand at an arbitrary point in time, was used to help determine an optimal operating point for a given system. Such was the case in experiment one when there appeared to be a number of acceptable operating points, (Q,R) pairs, for the system under investigation. This measure of performance was used in these cases to differentiate between these pairs.

As mentioned earlier, this model can also be used as a teaching aid as well as demonstrating analytical results obtained by others. Table XX (page 75) is a good example of this. If one sets the probability of repair equal to one, the model degenerates to the all repairables case without degradation. From the table, one can see how the various measures of performance change as the mean demand during repair time varies. Further, if Q is required to be one, the results of this model can be compared with the results of Feeney and Sherbrooke [Ref. 6]. Notice that in their results the fill rate was always zero when the reorder point was zero. This result was a consequence of their initial conditions, whereas the results obtained by this model do not include that restriction. If we again restrict the model by considering the all consumables case (i.e. letting the probability of repair be equal to zero) the model becomes that described by Hadley and Whitten [Ref. 3]. Using a sample problem, one again has an example of how the model can be used to verify known analytical results. This example and comparison to the cost approach is summarized in Table XXI.

Of course there are endless experiments one could carry out with the use of this model. For instance one could use this model to help decide which items should be designated repairable. This could be done by comparing the measures of performance obtained by considering an item first as a consumable and then as a repairable. It would also be easy

TABLE XX. COMPARISON BETWEEN FEENEY & SHERBROOKE'S [REF. 6] RESULTS WITH THIS

SIMULATIONS RESULTS

 \bar{X} =MEAN DEMAND DURING REPAIR TIME $Q=1$, $K=1$, $L=0$

REORDER POINT R	F(S)=FILL RATE				R(S)=READY RATE				P=1.0 (all repairables)			
	$\bar{X}=2.5$				$\bar{X}=1.50$				$\bar{X}=1.00$			
	F(S)	F&S R(S)	SIM F(S)	SIM R(S)	F(S)	F&S R(S)	SIM F(S)	SIM R(S)	F(S)	F&S R(S)	SIM F(S)	SIM R(S)
0	0	77.9	75.3	81.5	0	60.6	65.3	56.7	0	36.8	39.2	34.1
1	57.8	97.4	99.2	97.6	60.7	90.9	91.0	89.3	36.7	73.6	70.7	73.4
2	97.4	99.8	100	99.4	91.0	98.6	98.4	98.8	73.6	91.9	91.4	92.9
3	99.8	99.9	100	100	98.6	99.8	100	99.4	91.9	98.1	98.2	97.6
4	100	100	100	100	99.8	100	100	100	98.1	99.6	100	100
5	100	100	100	100	100	100	100	100	100	100	100	100
	%	%	%	%	%	%	%	%	%	%	%	%

TABLE XXI. COMPARISON BETWEEN HADLEY & WHITTEN [REF. 3] WITH THIS SIMULATION

Q=32, MEAN TIME BETWEEN DEMANDS = 3504 hours P=0.0 (all consumables)

	H&W	SIM
R	32	26
B _T	0	0

to extend this model to make use of the variance reduction techniques explained in reference nine or to add the capability of handling compound Poisson demands.

VII. SUMMARY AND CONCLUSION

The analysis of the repairable inventory problem presented in this paper was based upon a simulation model thought to be sufficiently realistic and general enough to represent many different inventory systems.

The results have shown that the simulation can be a helpful tool for the managers of inventory systems to use to make wise decisions governing their policies concerning when and how much to reorder. It was demonstrated in this paper that the mean time to repair an item, but not the repair time distribution itself, was extremely critical to system performance. It was demonstrated, at least for systems with a mean repair time less than or equal to 1.4, that a constraint placed on the minimum repair batch size had a negligible effect on system performance. It was further demonstrated for this same system that reducing the mean repair time by 40% was more profitable than doubling the maximum repair batch size. It also demonstrated that increasing maximum repair batch size beyond those values which gave a service rate of .417 or less had negligible effect in terms of increasing system performance. This paper also emphasized the power of using the expected number of unit years of backorders per unit period as the

criterion for judging system performance. This measure, when reduced to an acceptable level, was accompanied by good values of all other measures of performance as well. This is not true of the other measures. For example, if the fill rate had been used, Table XIII shows that an acceptable fill rate doesn't necessarily imply that all the other measures of performance are also acceptable.

It is believed that this paper supplements the tools which are needed to make intelligent decisions concerning the repairable item inventory system. Of course it can also be used for the consumables case by merely setting the probability of repair equal to zero.

A couple of further remarks seem to be in order. First, it would seem worthwhile to determine if current repair departments could be utilized more fully. Perhaps this could be achieved by spending a greater fraction of budget dollars to repair available carcasses rather than continuing to procure new material. Secondly, this study seems to indicate that study of a stock point's physical plant for repairables could be profitable. For example, the fourth experiment indicated that the minimum repair batch size required before repair begins on a batch of items could fluctuate to some degree before system performance is degraded.

Much has been learned about the interrelationships and effects of many of the parameters of repairable and non-repairable inventory systems. Perhaps this has been the

most significant result of the project. It would seem impossible for anyone who would work with this simulation not to come away with a better understanding of the hidden mechanisms and inner workings of an inventory system.

In closing, a simulation model for continuous review inventory systems was presented. The model used, demonstrated the sensitivity of Q, R, K, and L to numerous operational constraints and system parameters. A class of simple policies with various measures of performance was used rather than a traditional cost postulated model to determine acceptable parameters for a given inventory system. It is felt that the model can be successfully implemented in a very large number of inventory problems.

APPENDIX A. STATISTICAL TESTS

Analysis of variance. This test was used to find the significant difference in observed values of the expected number of unit years of backorders per unit period.

	Sum of Squares	Degrees of Freedom	Mean Square	F Ratio
Between groups	436.3584	11	39.6689	194.0706
Within groups	18.8052	92	0.2044	
Total	455.1636	103		

Table value from F-Distribution for 92 and 11 degrees of freedom at .01 significance level = 2.48.

$194.0706 > 2.48$, therefore the null hypothesis was rejected.

Null hypothesis: The observed values for the expected number of unit years of backorders per unit period are the same for each treatment (reorder point value).

K. A. Scheffe's Multiple Comparison Test. This test was used to find the magnitude required for the expected number of unit years of backorders per unit period to be significantly different from zero.

$$\text{If } \frac{(\sum |C_j|)^2}{(N-1)(\hat{V}(C_j))} > F_{(1-\alpha)(n_1)(n_2)}$$

then the hypothesis that the contrast C_j differs from zero is rejected.

$$C_j = \sum_{i=1}^t C_{ij} T_i$$

- $\hat{V}(C_j)$ is the estimated variance of contrast C_j .
- $\hat{V}(T_i)$ is the estimated variance of the i^{th} treatment = S^2 .
- T_i is the sum of the n_i observations for the i^{th} treatment.
- S^2 is the mean square of the experimental error.
- t is the number of treatments included in contrast C_j .
- C_{ij} are the coefficients for the j^{th} comparison with the i^{th} treatment.
- α is the significance level chosen = .01 for this particular test.
- $N-1$ is the total number of treatments less one.
- $(n_1)(n_2)$ are the degrees of freedom.

Let C_1 be the contrast between (QR) pairs (10,11) and (10,10) respectively. Then:

$$C_1 = C_{10,1} T_{10} + C_{11,1} T_{11} = (3)(-25.3176) + (-4)(-22.1442) = 12.62$$

$$\hat{V}(C_1) = \sum_{i=1}^{11} C_{i1}^2 (T_i) = (3^2)(8)(.2044) + (-4)^2(6)(.2044) = 34.339$$

$$\frac{(|C_1|)^2}{(N-1)(\hat{V}(C_1))} = \frac{(12.62)^2}{(11)(34.339)} = .422$$

.422 $\nless 2.48$ therefore, H_0 is not rejected.

Let C_2 be the contrast between (QR) pairs (10,11) and (10,9) respectively. Then:

$$C_2 = C_{9,2} T_9 + C_{11,2} T_{11} = (4)(22.404) + (-6)(-22.1442) = 43.24$$

$$\hat{V}(C_2) = (4^2)(9)(.2044) + (-6)^2(6)(.2044) = 73.584$$

$$\frac{(|C_2|)^2}{(N-1)(\hat{V}(C_2))} = \frac{(43.24)^2}{(11)(73.584)} = 2.309$$

2.309 $\nless 2.48$
Therefore, H_0
is not
rejected.

Let C_3 be the contrast between (QR) pairs (10,11) and (10,8) respectively. Then:

$$C_3 = C_{8,3} T_8 + C_{11,3} T_{11} = (4)(-14.353) + (-6)(-22.1442) = 75.453$$

$$\hat{V}(C_3) = (4)^2(9)(.2044) + (-6)^2(6)(.2044) = 73.584$$

$$\frac{(|C_3|)^2}{(N-1)(\hat{V}(C_3))} = \frac{(75.453)^2}{(11)(73.584)} = 6.994$$

6.994 > 2.48
Therefore,
reject H_0 , the
null hypothesis
that the means
are equal.

Let C_m be the contrast between (QR) pairs (10,11) and (10,m) respectively. Then:

$$\frac{(|C_m|)^2}{(N-1)(\sqrt{C_m})} = 2.48 = (|C_m|)^2 = (2.48)(11)(73.584) = 2007.372$$

$C_m = 44.8$ (minimum contrast value
required for statistical
significance.)

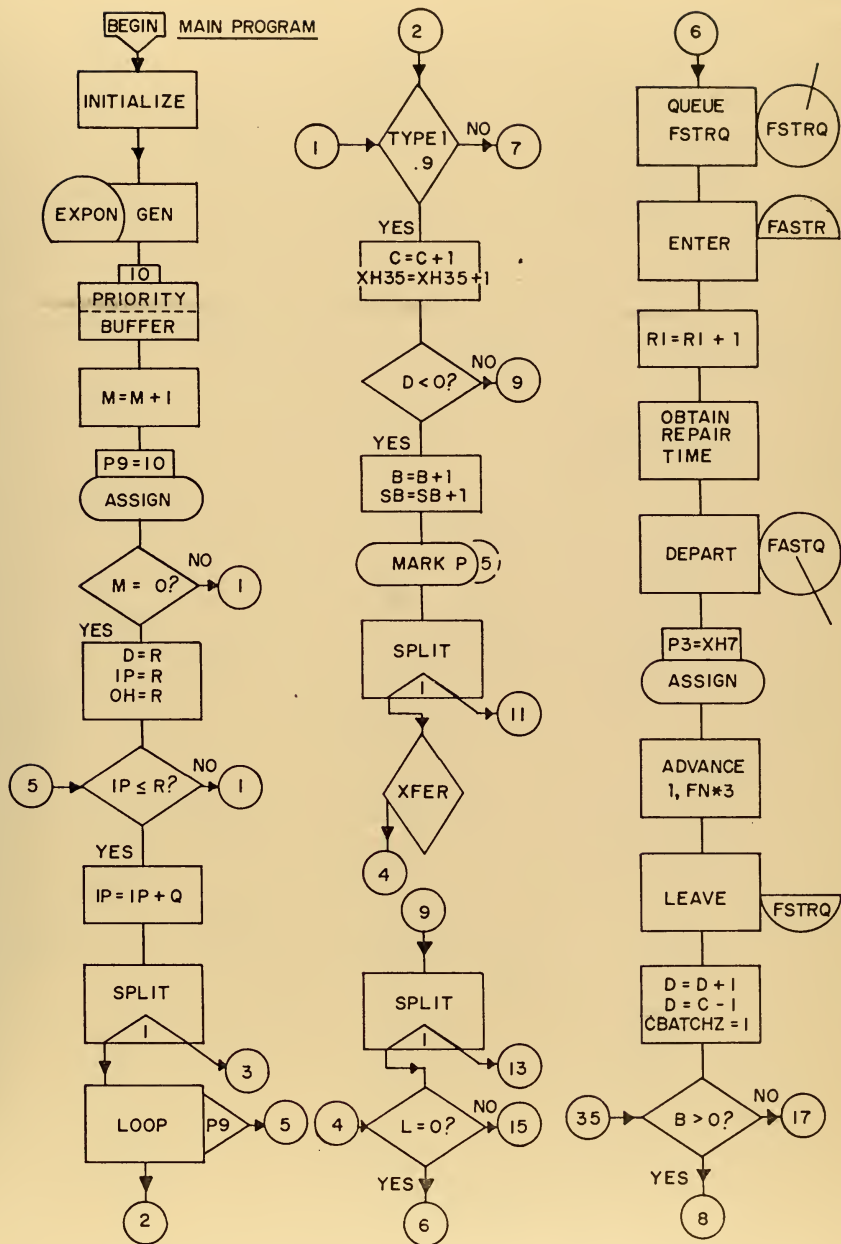
Then: $44.8 = C_{x,m}T_x + C_{11,m}T_{11} = (4)T_x + (-6)(22.404)$

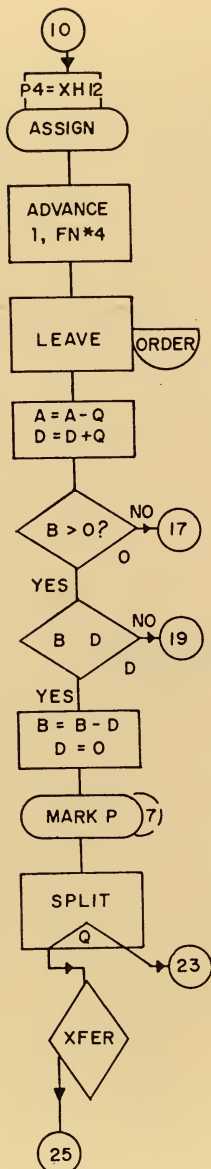
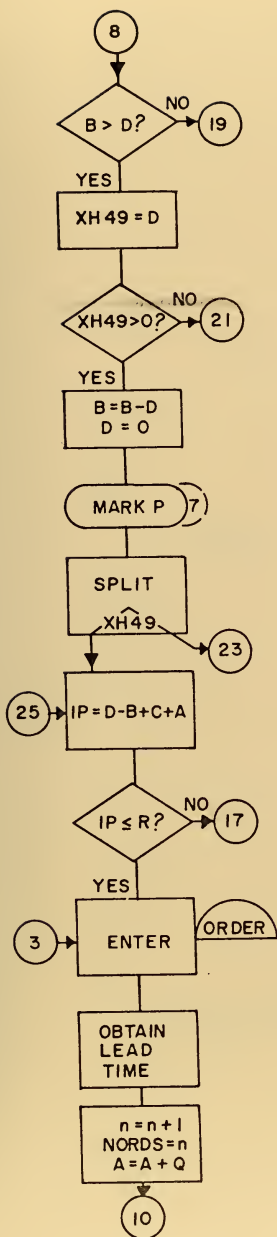
$$T_x = 22.016$$

$$\log_e B_T = -2.450$$

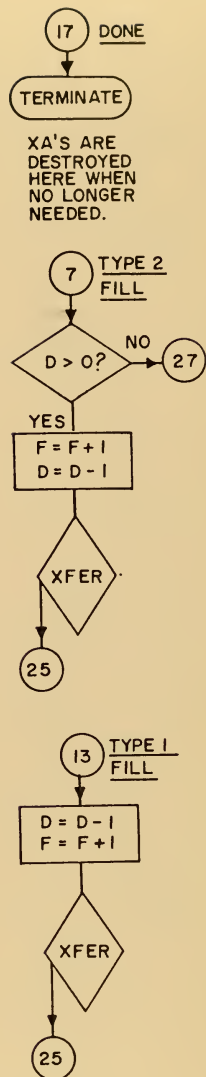
$B_T = .086$ the significant difference of the expected number
of unit years of backorders from .01

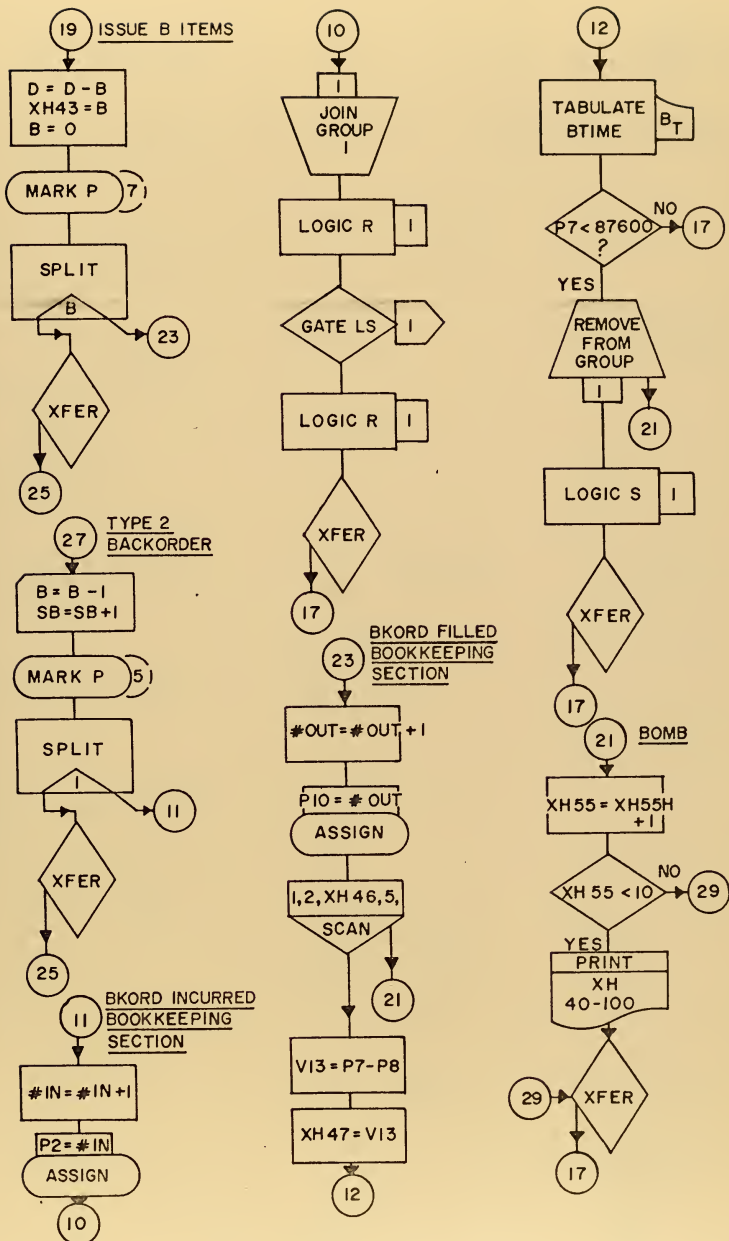
$B_T = .01 + .086 = 0.096$ the minimum significant difference
from zero

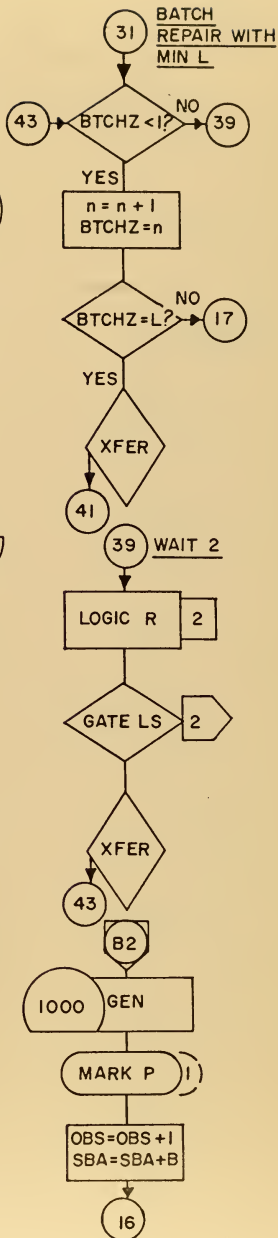
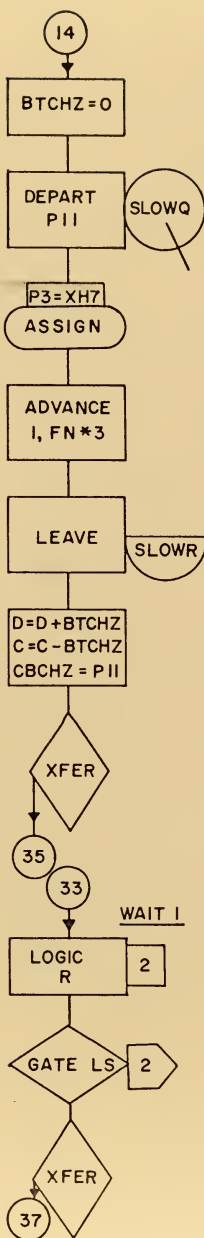
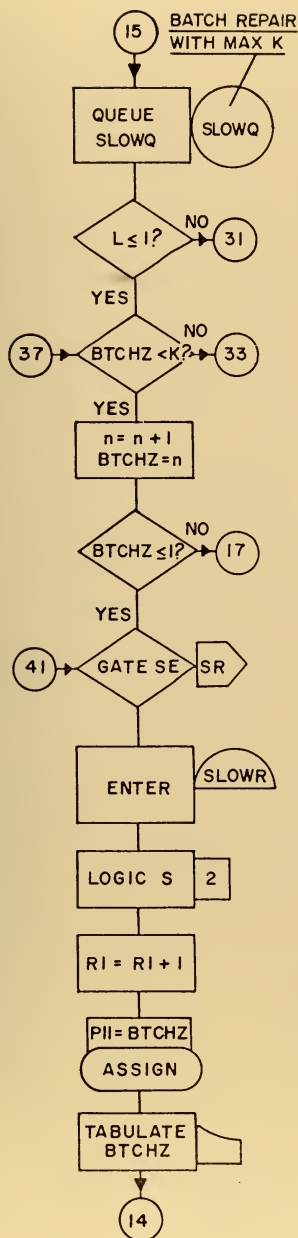


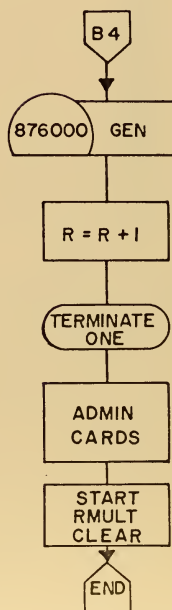
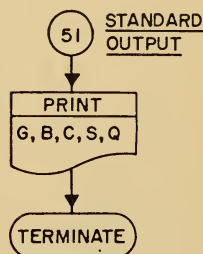
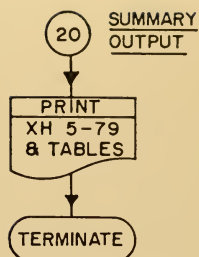
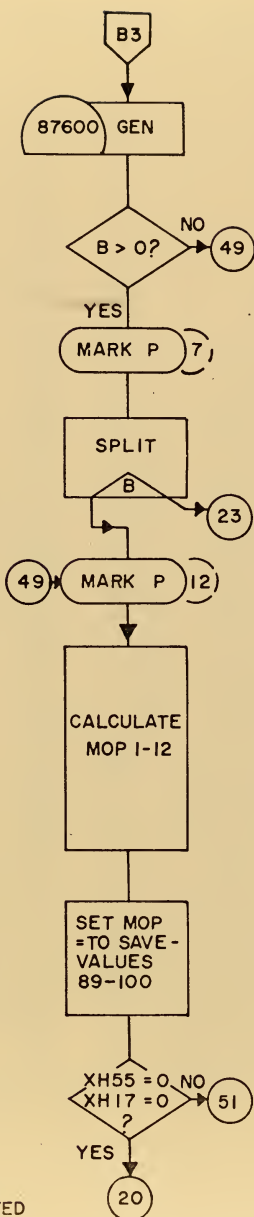
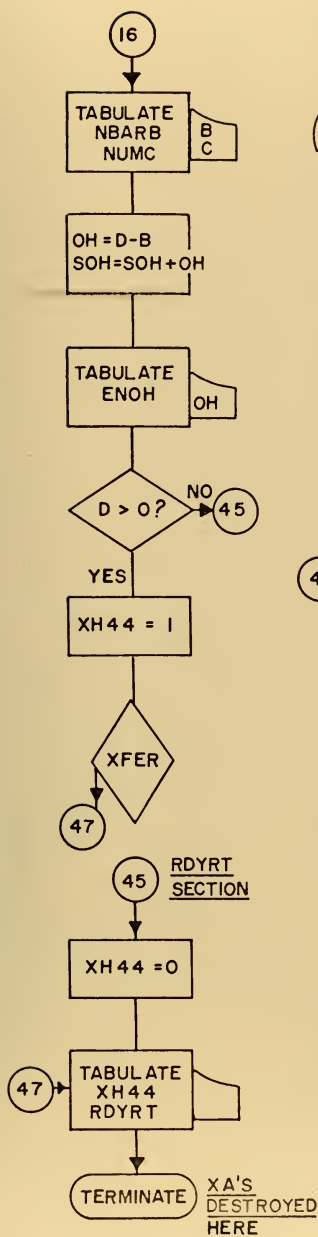


This completes the main part of the model.









APPENDIX C Program Listing

```

//THESE IS PROGRAM DECK
//JCLLIB DD DSN=GPSS, DISP=SHR
// EXEC GPSS
//DDOUT PUT DD SPACE=(CYL,8)
//GO.DINTERO DD SPACE=(CYL,(1,1))
//GO.DSYM TAB DD SPACE=(CYL,(1,1))
//GO.DREP TIG DD SPACE=(CYL,(1,1))
//GO.DREP IN DD *
//REALLOCATE BLO,200,FAC,0,SIO,4,QUE,3,LOG,2,FUN,12,VAR,20
//REALLOCATE PSV,5,HSV,100,CHA,0,BVR,0,FMS,0,HMS,0,COM,42600
//REALLOCATE TAB,06,GRP,1,XAC,500
//INITIAL XH1,10/XH2,01/XH3,0/XH4,001/XH5,0900/XH6,061/XH7,5
//INITIAL XH8,336/XH9,168/XH10,504/XH11,095/XH12,10/XH13,504
//INITIAL XH14,240/XH15,768/XH16,088/XH17,0
//SIMULATE
//SMULT 63785,32523,51497,20613,43315,13185,81207,96311
//GOING CONCERN OR GAME
*****
A PROBABILISTIC EVENT STORE COMPUTER SIMULATION OF THE INTERACTIONS
OF VARIOUS PARAMETERS OF A SINGLE ECULEON, CONTINUOUS REVIEW, QR,
REPAIRABLE ITEM INVENTORY SYSTEM.
*****
THE PURPOSE OF THIS MODEL IS TO TEST AND INVESTIGATE THE SENSITIVITY OF
THE PARAMETERS INVOLVED TO HELP THE DECISION MAKER EVALUATE OR SET UP
AN INVENTORY SYSTEM.
UNITS: ONE TIME UNIT = ONE HOUR. ONE UNIT PERIOD EQUALS 10 YEARS OF OP
EXPN FUNCTION RN8,C24
0/0,0,1,0,10470,2,0,2270,3,0,355/0,4,0,509/0,5,0,69/
0,6,0,1515/0,7,1,270,75,1,38/0,8,1,670,84,1,83/0,88,2,12/
0,9,2,3,0,92,1,52/0,94,2,81/0,95,2,99/0,96,3,27/0,97,3,5/
0,98,3,9/0,99,4,6/0,995,5,3/0,998,6,2/0,999,7,0/0,9997,8,0/
RN1M FUNCTION RN2,C2
0,0/1,0,1,0
3 FUNCTION RN4,E2
0,5,XH8/1,0,XH8
4 FUNCTION RN4,E2
0,5,XH036/1,0,XH036
5 FUNCTION RN4,E2
0,5,XH037/1,0,XH037
6 FUNCTION RN4,D10
0,20,2,15/0,40,230/0,50,235/0,60,290/0,80,450/
0,90,504/0,95,515/0,99,530/0,995,540/0,999,590/
7 FUNCTION RN3,E2
0,5,XH13/1,0,XH13
8 FUNCTION RN3,E2
0,5,XH038/1,0,XH038
CONSTANT REPAIR TIME
UNIFORM DISTN FOR REPAIR TIME
NORMAL DISTN FOR REPAIR TIME
EMPERC DISTN FOR REPAIR TIME
CONSTANT LEAD TIME.
UNIFORM DISTN FOR LEAD TIME.

```

2

13

4

6

10

20

30

40

50

60

70

80

90

100

110

160

170

180

190

200

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220

230

240

250

260

270

280

290

295

300

310

320

330


```

340 9 FUNCTION RN3,E2 NORMAL DIST'N FOR LEAD TIME.
350 10 FUNCTION RN3,D10 EMERCL DIST'N FOR LEAD TIME.
360 0.6,439/0.07,439/0.08,439/0.58,464/0.66,486
370 0.68,558/0.69,558/0.71,558/0.96,594/1.0,680
380 UNFMZ FUNCTION RN3,C2
390 0.0/1.0,100
400 NORMZ FUNCTION RN3,C69
410 0.0003,-340/0.0005,-330/0.0007,-320/0.0010,-310/0.0013,-300/0.0019,-290/
420 0.0026,-280/0.0035,-270/0.0047,-260/0.0062,-250/0.0082,-240/0.0107,-230/
430 0.0139,-220/0.0179,-210/0.0228,-200/0.0287,-190/0.0359,-180/0.0440,-170/
440 0.0548,-160/0.0668,-150/0.0808,-140/0.0968,-130/0.1151,-120/0.1357,-110/
450 0.1587,-100/0.1841,-90/0.2119,-80/0.2420,-70/0.2743,-60/0.3085,-50/
460 0.3446,-40/0.3821,-30/0.4207,-20/0.4602,-10/
470 0.5046,-10/0.5398,10/0.5793,20/0.6179,30/0.6554,40/0.6915,50/0.7257,60/
480 0.7580,70/0.7881,80/0.8159,90/0.8413,100/0.8643,110/0.8849,120/
490 0.9032,130/0.9192,140/0.9332,150/0.9452,160/0.9547,170/0.9641,180/
500 0.9713,190/0.9772,200/0.9821,210/0.9861,220/0.9891,230/0.9918,240/
510 0.9938,250/0.9953,260/0.9965,270/0.9974,280/0.9981,290/0.9987,300/
520 0.9990,310/0.9993,320/0.9995,330/0.9997,340
530 *
540 GENERATE XH6,FN$EXPON,...8..F GENERATE DMDS IAM PDISS DISTBN
545 PRIORITY 10,BUFFER ASSIGN HIGH PRI TO XA BEING PROCESSED
550 SAVEVALUE 079+*1,H DMDS=DMDS+1, TOT NUM OF DMDS T PERD
555 ASSIGN 9,10
560 TEST LE XH079,1,SPCT IF THIS ISN'T 1ST DMND GO TO SPCT
570 SAVEVALUE 074,XH2,H IF THIS IS FIRST DEMAND SET D=R
580 SAVEVALUE 077,XH2,H IF THIS IS FIRST DEMAND SET IP=R
590 TEST LE C57,XH2,H IF IP>R XPERXAR TO SPCT DON'T ORDER
600 SAVEVALUE XH070,XH2,SPCT IF IP>R XPERXAR TO SPCT DON'T ORDER
610 TEST LE 070+*XH1,H IF WEORD<R ORDER Q ITEMS. IP=IP+Q
620 SAVEVALUE 1,WEORD IF WEORD SEND XA TO ORDER. SECTION
630 SPLIT 9,ONE GO AND TEST IP LEVEL AGAIN.
640 LOOP INSPECT TO SEE IF ITEM TYPE 1 OR 2
650 TRANSFER XH5,TYPE2,TYPE1
660
670
680
690
700 THESE ARE TYPE ONE ITEMS (REPAIRABLE ITEMS).
710 TYPE1 073+*1,H C=C+1 C=NUM OF ITEMS IN REPAIR NOW.
720 SAVEVALUE 35+*1,H #TYPE I=#TYPE1+1.
730 TEST LE XH074,0,NEX1 IF D>0 DON'T BACKORDER. GO TO NEXT
740
750
760 *
770 *
780 *
790 *
800 *
810 *
820 *
830 *
840 *
850 *
860 *
870 *
880 *
890 *
900 *
910 *
920 *
930 *
940 *
950 *
960 *
970 *
980 *
990 *

```



```

** **
TEST LE V1,XH2,DONE IF IP>R KILL XA.
IP < OR = R A REORDER IS NECESSARY,
ORDER ENTER ORDER DEPT.
XH14+((XHX15-XH14)*FN$UNFMZ)/100)
038,V$UNFMX,H XH038$RANDOM X=U+(STD DEV)Z.
XH13+((XHX16*FN$NORMZ)/100)
NORMX UNIFORM LEAD TIME
039,V$NORMX,H XH039$RANDOM NORMAL LEAD TIME.
SAVEVALUE 075+,I,H THE # OF ORDS T RUN.
SAVEVALUE 072+,XH1,H A=A+Q,A=NUM ON ORDER NOW.
SAVEVALUE 4,XH12 ASS TYPE OF LEAD TIME DIST*BN TO P4.
ADVANCE 1,FN#4 ORDER Q ITEMS.
LEAVE RECEIVE ORDER.
SAVEVALUE 072-,XH1,H A=A-Q.
SAVEVALUE 074+,XH1,H D=D+Q.

AN ORDER HAS BEEN RECEIVED CHECK TO SEE IF AN ISS IS REQUIRED.

** **
TEST G XH071,0,DONE IF B=0, KILL XA.
B>0, AN ISS WILL FOLLOW.

TEST GE XH071,XH074,ISS/B IF B<D ISS B ITEMS.SEND XA TO ISS/B.
B > OR = D, ISSUE D ITEMS.

SAVEVALUE 071-,XH074,H B=B-D.
SAVEVALUE 074,0,H D=0.
MARK ISSUE D ITEMS.
SPLIT TIME THESE D BKORD'S FILLED
TRANSFER SEND D XA'S TO B BOOKKEEPING SECTION.
TERMINATE CHECK TO SEE IF REORD NECESSARY
DONE XA'S ARE KILLED HERE.

** **
THIS COMPLETES THE MAIN PART OF THE MODEL

THE BRANCHES FROM THE ABOVE PROGRAM FOLLOW

THIS IS A TYPE2 ITEM (NON REPAIRABLE) SEE IF WE CAN FILL DEMAND.
TYPE2 TEST G XH074,0,BKORD IF D=0 SEND XA TO BKORD SECTION
** **

```

1200
1205
1210
1220
1230
1240
1250
1260
1270
1280
1290
1300
1310
1320
1330
1340
1350
1355
1360
1365
1370
1380
1390
1400
1410
1420
1430
1440
1450
1460
1470
1480
1490
1500
1510
1520
1530
1540
1550
1555
1560
1570
1580
1584
1585


```

1590
1595
1596
1600
1610
1620
1630
1635
1636
1640
1650
1660
1670
1680
1690
1695
1700
1710
1720
1730
1740
1750
1760
1770
1780
1785
1790
1800
1810
1820
1830
1840
1850
1860
1865
1870
1880
1890
1900
1910
1920
1930
1940
1950
1960
1965
1970
1980

***
D>O WE CAN FILL DEMAND IMMEDIATELY.

SAVEVALUE 077+,1,H
SAVEVALUE 074-,1,H
TRANSFER ,ORDER
FILL DEMAND (ISS ITEM!., F=F+1.
D=D-1.
IS A REORDER NEC? SEND XA TO ORDER

TYPE 1 ITEM WITH D>O WE CAN FILL DEMAND IMMEDIATELY.
ISUE1 SAVEVALUE 074-,1,H
SAVEVALUE 077+,1,H
TRANSFER ,ORDER
ISSUE ITEM.
F=F+1, F=NUM OF FILLS THIS RUN,
IS A REORDER NEC? SEND XA TO ORDER

AN ORDER OR REPAIRED ITEM WAS RECEIVED AND B<D SO ISSUE B ITEMS.
ISS/B SAVEVALUE 074-,XH071,H
SAVEVALUE 043,XH071,H
SAVEVALUE 071,0,H
MARK 7
SPLIT XH043,GROP1
TRANSFER ,ORDER
D=D-B.
RECORD # OF ITEMS RECEIVED IN XH43
B=0.
RECORD TIME B BRORD'S FILLED.
SEND B XA TO B BOOKKEEPING SECTION
CHECK TO SEE IF REORD NECESSARY.

TYPE 2 DEMANDS WITH D=0 SO THEY ARE BACKORDERED.
BKORD SAVEVALUE 071+,1,H
SAVEVALUE 098+,1,H
MARK 5
SPLIT RECORD TIME OF B.
TRANSFER 1,GRUP1
ORDER SEND XA TO B BOOKKEEPING SECTION.
CHECK TO SEE IF REORD NECC

BACKORDER INCURRED BOOKKEEPING SECTION
GRUP1 SAVEVALUE 045+,1,H
ASSIGN 2,XH045
JOIN INDEX B INCURRED THIS RUN
LOGIC R FLAG XA AS BEING B.
GATE LS RESET GATE SO B WAITS TIL FILLED
LOGIC R ALLOW XA THROUGH IF B FILLED
TRANSFER 1,DONE CLOSE GATE, NEXT B WAITS TIL FILL

BACKORDER FILLED BOOKKEEPING SECTION

```



```

1990
1991
2000
2010
2020
2030
2040
2050
2060
2070
2080
2090
2100
2110
2120
2130
2140
2150
2160
2170
2180
2190
2200
2210
2220
2230
2240
2250
2260
2270
2280
2290
2300
2310
2320
2330
2340
2350
2360
2370
2380
2390
2400
2410
2420

```

* GROUPL SAVEVALUE 046+,1,H #OUT=#OUT+1,# OF B FILEDTHIS RUN
* ASSIGN 10,XH046 ASSIGN INDEX OF FILLED B TO PLO.
* SCAN 1,2,XH046,5,8,BOMB GET OLDEST B AND PUT TIM B IN P8

IF NO B FOUND WITH THIS INDEX THERE IS AN ERROR SO BOMB
13 VARIABLE P7-P8 FIND TIME THIS B OUTSTANDING
SAVEVALUE 047,V13,H RECORD TIME IN XH047,
TABULATE BITIME TIME IN TABLE BTIME.
TEST L IF P7=87600,DONE IF P7=87600,KILL XA
REMOVE 1,1,,2,XH046,BOMB REMOVE THIS B FROM GRUP 1.
LOGIC S 1,B FILLED, SO OPEN GATE IN GRUP 1
TRANSFER ,DONE

IF A XA GETS HERE YOU HAVE MADE AN ERROR IN YOUR INPUT, CHECK IT.
BOMB SAVEVALUE 055+,1,H BOMB=BOMB+1 BOMB=NUM OF ERRORS.
TEST L XH55,10,GET TOT
PRINT 40,100,XH,D KILL XA
GETTOT TRANSFER ,DONE

THIS SECTION HANDLES THE BATCH REPAIR CASE.
BATCH ADVANCE O SLOWQ ITEM ENTERS REPAIR DEPT
QUEUE SLOWX H3,1,MINBZ IF WE REQ MIN BATCHES GO TO MINBZ
TEST LE K>1, L=1 (RESTRICTION ON MAX BTCHZ ONLY)
BATCH REPAIR, IS A ONE SERVER QUEUE REPAIR OPERATION WITH
ONLY ONE ITEM BEING REPAIRED AT A TIME.
RULE1 TEST L XH048,XH4,WAITI IF BTCHZ=K & REP DEPT BUSY,WAITI
SAVEVALUE 048+,1,H BTCHZ=BTCHZ+1
TEST LE XH048,1,DONE IF BTCHZ>1 KILL XA

THIS ENSURES ONLY ONE XA ENTERS REP DEPT WHILE XH048(BTCHZ)
IS,((L-1) < BTCHZ < (K+1))

RULE3 GATE SE SLOWR IF REP DEPT IDLE,ENTER:ELSE WAIT
ENTER SLOWR BEGIN REPAIR ON NEW BATCH
LOGIC S NEXT BTCH IN REP DEPT,FORM NEW BTCH
SAVEVALUE 076+,1,H NUMR1=NUMR1+1 # OF REPAIR INDUCTS
ASSIGN 11,XH048 P11=CURRENT BATCH SIZE (BTCHZ)
TABULATE BTCHZ RECORD BTCHZ

```

SAVEVALUE 048,0,H
DEPART SLOWQ,P11
ASSIGN 3,XH7
ADVANCE 1,FN*3
LEAVE SLOWR
SAVEVALUE 074+,P11,H
SAVEVALUE 073-,P11,H
SAVEVALUE 049,P11,H
TRANSFER ,ISUEZ
2430
2440
2450
2460
2470
2480
2490
2500
2510
2520
2530
2540
2550
2560
2570
2580
2590
2600
2610
2620
2630
2640
2650
2660
2670
2680
2685
2690
2700
2710
2720
2725
2730
2740
2750
3070
2770
2780
2790
2800
2810
2815
2816

***
WAIT1 LOGIC R
GATE LS
TRANSFER ,RULE1
2
MINBZ REPAIR, K>1, L>1 (MIN & MAX REQ ON BTCHZ)
2605
2610
2620
2630
2640
2650
2660
2670
2680
2685
2690
2700
2710
2720
2725
2730
2740
2750
3070
2770
2780
2790
2800
2810
2815
2816

***
MINBZ RULE2
ADVANCE XH048,XH4,WAIT2
TEST L XH048+,1,H
SAVEVALUE XH048,XH3,DONE
TEST E ,RULE3
TRANSFER ,RULE3
2610
2620
2630
2640
2650
2660
2670
2680
2685
2690
2700
2710
2720
2725
2730
2740
2750
3070
2770
2780
2790
2800
2810
2815
2816

***
WAIT2 LOGIC R
GATE LS
TRANSFER ,RULE2
2
REP DEPT BUSY & THE NEX BTCHZ=K SO THESE ITEMS MUST WAIT
TIL THE REP DEPT IS FREE BEFORE FORMING A NEW BATCH
2670
2680
2685
2690
2700
2710
2720
2725
2730
2740
2750
3070
2770
2780
2790
2800
2810
2815
2816

***
THIS COMPLETES THE BATCH REPAIR PART OF THE MODEL
2730
2740
2750
3070
2770
2780
2790
2800
2810
2815
2816

NOW COMPUTE THE RANDOM OBS TIMES AND TABULATE OBS,BA,C,AND OH.
2730
2740
2750
3070
2770
2780
2790
2800
2810
2815
2816

GENERATE 1000,FN$RNTIM,,,,F CREATE RANDOM OBSERVATION TIMES.
MARK 1 RECORD TIME FOR NEXT OBS IN PHS
SAVEVALUE 059+,1,H
SAVEVALUE 056+,XH071,H
VARIABLE XH71*100
6 SAVEVALUE 54,V6,H
SBA=SBA+B, SBA=NUM OF BA THIS YR.
SBS=SBS+1, SBS=NUM OF OBS THIS YR.

```



```

TABULATE NBARB
VARIABLE XH73*100
SAVEVALUE 53,V7,H
TABULATE (XH074-XH071)*100
VARIABLE 058,V2,H
SAVEVALUE ENOH
TABULATE XH074,0,MOVE
TEST G 044,10000,H
SAVEVALUE TAB
TRANSFER TAB
SAVEVALUE 044,0,H
TABULATE RDYRT
MOVE
TAB

**
*****

TERMINATE

CONTROL OF RUNNING TIME AND CALCULATION OF MEASURES OF PERFORMANCE.

87600,0,5,F
XH71,0,NOBL
MARK
TEST G
SPLIT
MARK
NOBL 3
SAVEVALUE 001,P12
SAVEVALUE XH074-XH071+XH073+XH072
VARIABLE 070,V3,H
SAVEVALUE (XH2+(XH1+1)/2-(XH5/1000)*(1/XH6)*XH8)*100
VARIABLE V5-((1-XH5/1000)*(1/XH6)*XH13)*100
FVARIABLE ((TB$BTIME*TC$BTIME)/(8760)*100
MOP4 FVARIABLE 097,V$MOP4,H
SAVEVALUE (TB$ENOH/V4)*100
RAT01 FVARIABLE 088,V$RAT01,H
SAVEVALUE (V$MOP4/10-TB$NBARB
FVARIABLE (V$MOP4/10)/(TB$NBARB)*100
RAT02 FVARIABLE 087,V$RAT02,H
SAVEVALUE (XH077/XH079)*10000
FILRT FVARIABLE 069,V4,H
SAVEVALUE TB$ENOH-V4
DIFF1 FVARIABLE 090,V$DIFF1,H
SAVEVALUE (XH056/XH059)*100
BARB FVARIABLE 96,V$BARB,H
SAVEVALUE 95,TB$RDYRT,H
SAVEVALUE 089,V$DIFF2,H
SAVEVALUE 087,V$RAT02,H
SAVEVALUE 099,V$FILRT,H
SAVEVALUE 091,TB$NUNC,H
SAVEVALUE 078,XH098,H

RECORD BA, BA=NUM OF B NOW.

RECORD C, NUM IN REPAIR DEPT NOW.
V2=OH=D-B, NUM ON HAND NOW.
SOH=SOH+OH, SOH=# GH THIS RUN*100.
RECORD # ON HAND NOW
IF D=0 SEND XA TO MOVE
XH044=1000 => D > 0
SEND XA TO TAB BLOCK
IF D=0, XH044=0
TAB XH44,XH44=1 IF D > 0, 0 IF D=0

THIS COMPLETES THE TABULATION PHASE.

3320
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3990
4000
4010
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4090
4100
4110
4120
4130
4140
4150
4160
4170
4180
4190
4200
4210
4220
4230
4240
4250
4260
4270
4280
4290
4300
4310
4320
4330
4340
4350

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SAVEVALUE 092,XH076,H
SAVEVALUE 093,XH075,H
SAVEVALUE 078,XH098,H
SAVEVALUE 100,XH77,H
SAVEVALUE 94,TB$ENDH,H
TEST E,XH55,0,JUMP
TEST E,XH17,0,JUMP
PRINT I,5,100,XH,D
PRINT I,T
PRINT I,T
JUMP 3360
JUMP 3370
JUMP 3380
JUMP 3390
JUMP 3400
JUMP 3404
JUMP 3450
JUMP 3451

```

TERMINATE	GENERATE	87600	INCREMENT	R BY ONE	
SAVEVALUE	2+, 1, H				3460
TABLE	1	XH054, 0, 100, 25			3470
TABLE		XH044, 0, 10000, 3			3480
TABLE		XH53, 0, 100, 25			3490
ENCL		XH058, -500, 500, 25	TB OF TABLE =	READY RATE, MOP #9	3500
TABLE		XH047, 50, 50, 25			3510
TABLE		XH048, 1, 1, 25			3520
TABLE		999999	BORD TIMES		3530
STORAGE		999999	BICH SIZE		3540
STORAGE		999999			3550
STORAGE		999999			3560
FASTER		999999			3570
SLOWER		999999			3580
START	1				3590
					3600

```

30, I. SAVEVALUES (SYSTEM PARAMETERS & STATISTICS)
5, 3
3 SPACE
3 TITLE
5, A. CLOCK TIME WHEN STATISTICS WERE GATHERED.
2 TITLE
2 SPACE
10 TEXT
UN SIMULATES
SPACE
HSVAV TITLE
10 SPACE
TEXT ORDER QUANTITY = #XH1.2/XXX#
10 SPACE
TEXT REORDER POINT '= #XH2.2/XXX#
10 SPACE
TEXT K, MAXIMUM BATCH SIZE = #XH4.2/XXX#
10 SPACE
TEXT L, MINIMUM BATCH SIZE = #XH3.2/XXX#
10 SPACE

```


30, C. MEASURES OF PERFORMANCE, (MOP).
 SPACE TITLE
 10 MOP 1 = #XH100,2/XXXXXX# , EXP NUM OF FILLS.
 TEXT
 10 MOP 2 = #XH99,2/2LXX.XX#% , FILL RATE
 TEXT
 10 MOP 3 = #XH98,2/XXXXX# , EXP NUM OF BACKORDERS.
 TEXT
 10 MOP 4 = #XH97,2/2LXX.XX# , EXPECTED NUMBER OF UNIT C
 TEXT YEARS OF BACKORDERS PER UNIT PERIOD (10 YRS)
 SPACE
 10 MOP 5 = #XH96,2/2LXX.XX# , EXP NUM OF BACKORDERS ATC
 TEXT AN ARBITRARY POINT IN TIME.
 SPACE
 10 MOP 6 = #XH95,2/2LXX.XX#% , READY RATE, THE PROBABILC
 TEXT ITY OF A FILL.
 SPACE
 10 MOP 7 = #XH94,2/2LXX.XX# , EXP NUMBER OH-HAND (OH) C
 TEXT AT AN ARBITRARY POINT IN TIME.
 SPACE
 10 MOP 8 = #XH93,2/XXXXXX# , EXP NUM OF ORDERS PLACED.
 TEXT
 10 MOP 9 = #XH92,2/XXXXXX# , EXP NUM OF REP INDUCTIONS
 TEXT
 10 MOP 10 = #XH91,2/2LXX.XX# , EXP NUM IN REPAIR (C) AT C
 TEXT AN ARBITRARY POINT IN TIME.
 SPACE
 10 MOP 11 = #XH90,2/2LXX.XX# , DIFF1 OH(SIM)-OH(THEORY)
 TEXT
 10 MOP 12 = #XH89,2/2LXX.XX# , DIFF2 MOP4(S)-MOP5(SIM)
 TEXT
 10 MOP 13 = #XH88,2/XXXXXX.XX#% , RATIO1 OH(SIM)/OH(THEORY)
 TEXT
 10 MOP 14 = #XH87,2/XXXXXX# , RATIO2 MOP4(S)/MOP5(S).
 TEXT
 4, II. STORAGE STATISTICS.
 3
 STO SPACE
 2, A. STATISTICS FOR ORDER DEPARTMENT.
 STO SPACE
 1, B. STATISTICS FOR FAST REPAIR PROCESS. C
 TITLE SERVER QUEUE REPAIR SYSTEM).
 2, C. STATISTICS FOR SLOW REPAIR PROCESS. C
 STO (FINITE SERVER QUEUE REPAIR SYSTEM)
 SPACE
 3, III. QUEUE STATISTICS.
 3, TITLE
 3, SPACE

QUE
QUE

TITLE
SPACE
TITLE
END

1,
2,
2,

A. STATISTICS FOR REPAIR DEP (INFINITE SYSTEM).
B. STATISTICS FOR REPAIR DEPT (FINITE SYSTEM).

3610

APPENDIX D SUMMARY AND STANDARD OUTPUT

CONTENTS OF HALFORD SAVEVALUES (NON-ZERO)		NR. VALUE		NR. VALUE		NR. VALUE		NR. VALUE		NR. VALUE		NR. VALUE	
SAVEVALUE	NR. VALUE	NR. VALUE	NR. VALUE	NR. VALUE	NR. VALUE	NR. VALUE	NR. VALUE	NR. VALUE	NR. VALUE	NR. VALUE	NR. VALUE	NR. VALUE	NR. VALUE
13	504	14	240	15	768	16	336	17	168	18	504	19	1008
39	446	43	4	44	10000	45	685	46	685	47	153	48	153
56	186	57	821	58	100	59	169	60	101	61	101	62	101
91	1528	92	1366	93	13	94	173	95	5384	96	110	97	1157
99	5472	100	1828										

I. SAVEVALUES (SYSTEM PARAMETERS & STATISTICS)

A. CLOCK TIME WHEN STATISTICS WERE GATHERED.

87600 = THE NUMBER OF HOURS OF OPERATION THIS RUN SIMULATES

B. CRITICAL SYSTEM PARAMETERS.

Q, ORDER QUANTITY = 10
 R, REORDER POINT = 2
 K, MAXIMUM BATCH SIZE = 1
 L, MINIMUM BATCH SIZE =

C. MEASURES OF PERFORMANCE, (MOP).

MOP 1 = 828 , EXP NUM OF FILLS.
 MOP 2 = 54.72% , FILL RATE
 MOP 3 = 685 , EXP NUM OF BACKORDERS.
 MOP 4 = 11.57 , EXPECTED NUMBER OF UNIT YEARS OF BACKORDERS PER UNIT PERIOD (10 YRS)
 MOP 5 = 1.10 , EXP NUM OF BACKORDERS AT AN ARBITRARY POINT IN TIME.
 MOP 6 = 53.84% , READY RATE, THE PROBABILITY OF A FILL.

MOP 7 = .73 , EXP NUMBER OH-HAND (OH) AT AN ARBITRARY POINT IN TIME.
 MOP 8 = 15 , EXP NUM OF ORDERS PLACED.
 MOP 9 = 1366 , EXP NUM OF REP INDUCTIONS
 MOP 10 = 5.28 , EXP NUM IN REPAIR (C) AT AN ARBITRARY POINT IN TIME.
 MOP 11 = .01 * DIFF1 OH(SIM)-OH(THEORY)
 MOP 12 = .05 * DIFF2 MOP4(S)-MOP5(SIM)
 MOP 13 = 101.0 % RATIO1 OH(SIM)/OH(THEORY)
 MOP 14 = 105% , RATIO2 MOP4(S)/MOP5(S).

II. STORAGE STATISTICS.

A. STATISTICS FOR ORDER DEPARTMENT.

STORAGE ORDER	CAPACITY	AVERAGE CONTENTS	AVERAGE UTILIZATION	ENTRIES	AVERAGE TIME/TRANS	CURRENT CONTENTS	MAXIMUM CONTENTS
	99999	.006	.000	15	508.086		1

B. STATISTICS FOR FAST REPAIR PROCESS.

STORAGE FASTR	CAPACITY	AVERAGE CONTENTS	AVERAGE UTILIZATION	ENTRIES	AVERAGE TIME/TRANS	CURRENT CONTENTS	MAXIMUM CONTENTS
	99999	5.226	.000	1366	335.171		15

(INFINITE SERVER QUEUE REPAIR SYSTEM).

C. STATISTICS FOR SLOW REPAIR PROCESS.

(FINITE SERVER QUEUE REPAIR SYSTEM)

III. QUEUE STATISTICS.

A. STATISTICS FOR REPAIR DEP (INFINITE SYSTEM).

QUEUE	MAXIMUM CONTENTS	AVERAGE CONTENTS	TOTAL ENTRIES	ZERO ENTRIES	PERCENT ZEROS	AVERAGE TIME/TRANS	AVERAGE TIME/TRANS	TABLE NUMBER	CURRENT CONTENTS
ESTRO	1366	.000	1366	1366	100.0	.000	.000		
\$AVERAGE TIME/TRANS = AVERAGE TIME/TRANS EXCLUDING ZERO ENTRIES									

B. STATISTICS FOR REPAIR DEPT (FINITE SYSTEM).

END

STANDARD OUTPUT ONLY

TABLE B TIME ENTRIES IN TABLE 665									
UPPER LIMIT	OBSERVED FREQUENCY	MEAN ARGUMENT 148.417	STANDARD DEVIATION 92.625	SUM OF ARGUMENTS 101666.000	NON-WEIGHTED				
					DEVIATION FROM MEAN	MULTIPLE OF	CUMULATIVE REMAINDER	CUMULATIVE PERCENTAGE	
150	120				-.279		82.4	56.3	
160	127				-.010		46.1	53.8	
170	117				1.010		29.0	70.9	
200	117				1.347		16.0	83.3	
250	69				1.684		6.5	93.4	
300	35				2.021		1.4	98.5	
350	10				2.358		100.0		
400	10				2.695				
REMAINING FREQUENCIES ARE ALL ZERO									
TABLE NBARS ENTRIES IN TABLE 169									
UPPER LIMIT	OBSERVED FREQUENCY	MEAN ARGUMENT 110.059	STANDARD DEVIATION 205.437	SUM OF ARGUMENTS 18500.000	NON-WEIGHTED				
					DEVIATION FROM MEAN	MULTIPLE OF	CUMULATIVE REMAINDER	CUMULATIVE PERCENTAGE	
0	119				-.008		27.5	72.7	
100	14				1.817		2.2	83.4	
200	18				3.725		16.5	89.3	
300	10				5.634		6.5	95.8	
400	4				7.543		1.1	97.6	
500	3				9.451		2.2	98.8	
600	2				11.360		1.1	99.4	
700	2				13.268		1.1	99.8	
800	0				15.177		1.5	100.0	
900	0								
1000	0								
1100	1								
1200	0								
1300	1								
REMAINING FREQUENCIES ARE ALL ZERO									

TABLE ENTRIES IN TABLE	NUMC	MEAN	ARGUMENT	STANDARD DEVIATION	SUM OF ARGUMENTS	NON-WEIGHTED
169			528.993	216.375	85200.000	
UPPER LIMIT	0	OBSERVED FREQUENCY	PER CENT OF TOTAL	CUMULATIVE PERCENTAGE	CUMULATIVE REMAINDER	MULTIPLE OF MEAN
100	1	1	1.77	5	97.9	DEVIAION FROM MEAN
200	127	127	1.59	2.3	99.4	-2.3444
300	17	17	1.00	19.4	98.5	-1.5822
400	33	33	1.00	36.2	97.9	-1.1028
500	33	33	1.00	56.2	93.7	-1.1028
600	31	31	1.00	74.5	63.7	-1.1028
700	16	16	1.00	84.0	43.7	-1.1028
800	16	16	1.00	93.4	15.9	1.1028
900	3	3	1.77	97.0	1.512	1.1028
1000	0	0	1.77	98.8	1.701	1.1028
1100	0	0	1.59	98.8	1.870	1.1028
1200	1	1	1.59	100.0	2.2457	1.1028
REMAINING FREQUENCIES ARE ALL ZERO						3.563

TABLE ENTRIES IN TABLE	NUMC	MEAN	ARGUMENT	STANDARD DEVIATION	SUM OF ARGUMENTS	NON-WEIGHTED
169			73.964	372.000	12500.000	
UPPER LIMIT	0	OBSERVED FREQUENCY	PER CENT OF TOTAL	CUMULATIVE PERCENTAGE	CUMULATIVE REMAINDER	MULTIPLE OF MEAN
1000	0	0	1.18	1.1	100.0	DEVIAION FROM MEAN
2000	0	0	1.18	1.1	100.0	-13.4395
3000	0	0	1.18	1.1	100.0	-10.9551
4000	0	0	1.18	1.1	100.0	-8.5071
5000	0	0	1.18	1.1	100.0	-8.5071
6000	0	0	1.18	1.1	100.0	-5.275
7000	0	0	1.18	1.1	100.0	-5.275
8000	0	0	1.18	1.1	100.0	-2.887
9000	2	2	5.32	6.5	98.8	-1.542
10000	97	97	39.24	46.1	53.4	-1.198
11000	77	77	45.56	91.7	8.2	1.145
12000	14	14	8.28	100.0	13.520	2.445
REMAINING FREQUENCIES ARE ALL ZERO						

TABLE ENTRIES IN TABLE	NUMC	MEAN	ARGUMENT	STANDARD DEVIATION	SUM OF ARGUMENTS	NON-WEIGHTED
169			358.813	4992.000	91000.000	
UPPER LIMIT	0	OBSERVED FREQUENCY	PER CENT OF TOTAL	CUMULATIVE PERCENTAGE	CUMULATIVE REMAINDER	MULTIPLE OF MEAN
10000	0	0	46.15	100.0	53.8	DEVIAION FROM MEAN
11000	78	78	46.15	100.0	8.2	-1.078
12000	91	91	53.84	100.0	1.857	.924
REMAINING FREQUENCIES ARE ALL ZERO						

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14. KEY WORDS

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Q73 Quirk

c.1

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step computer simulation
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inventory system.

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